St John Bosco
Maths Curriculum Policy and Strategies




The Aims of our Curriculum Policy

This policy has been written in accordance with the National Curriculum 2014 and to support the three main aims of fluency, mathematical reasoning and problem solving. It is designed to provide pupils with a consistent and fluent progression of learning when using the four main operations.

The calculation policy is organised according to age related expectations as set out in the National Curriculum 2014, however it is vital that pupils are taught according to the stage that they are currently working at, moving on when they are secure. Decisions about when to progress should always be based on the security of pupils' understanding. Pupils who grasp concepts rapidly should be challenged through being offered rich and sophisticated problems before any acceleration through new content. Those who are not sufficiently fluent with earlier concepts should consolidate their understanding, through additional practice, before moving on.

It is important that any type of calculation is given a real life context or problem solving approach to help build children's understanding of the purpose of calculations, and to help them recognise when to use certain operations and methods when faced with problems. The priority in every maths lesson and the importance of teaching maths across the curriculum is to allow the children to use and apply their calculation skills.

Pedagogical Approaches

The aims of the curriculum for mathematics at St John Bosco:

- Developing pupils' understanding of number and place value is essential and should be explored daily.
- The strategies chosen should aim to develop pupils' conceptual understanding of calculation.
- Models, images and resources (representations) should be used throughout all key stages.
- Pupils should be encouraged to develop independence, and to select and use resources to support their learning.
- Practical activities should be a regular feature of maths lessons.
- Activities should be differentiated to suit the needs of the pupils.
- Opportunities to work within mixed ability groups should be explored.
- It is more effective to provide pupils with one question to practise the same skill rather than lots of different questions.
- Solving problems should be integral to the maths curriculum.
- Pupils should be encouraged to take risks, make mistakes, and learn from their experiences.
- Teachers will explore misconceptions with pupils in order to deepen their understanding.

Fluency, Reasoning and Problem Solving

What does fluency, reasoning and problem solving look like in solving calculation questions?

These are the three aims from the 2014 Mathematics National Curriculum which are to ensure all pupils:

- become fluent in the fundamentals of mathematics, through varied and frequent practice with increasingly complex problems over time, so that pupils develop conceptual understanding and the ability to recall and apply knowledge rapidly and accurately

- reason mathematically by following a line of enquiry, conjecturing relationships and generalisations, and developing an argument, justification or proof using mathematical language

- can solve problems by applying their mathematics to a variety of routine and non-routine problems with increasing sophistication, including breaking down problems into a series of simpler steps and persevering in seeking solutions.

The 2014 mathematics curriculum states that 'Mathematics is an interconnected subject in which pupils need to be able to move fluently between representations of mathematical ideas… (all) pupils should make rich connections across mathematical ideas to develop fluency, mathematical reasoning and competence in solving increasingly sophisticated problems.'

Fluency, Reasoning and Problem Solving

Examples of fluency, reasoning and problem solving:

$8 \times 5 = 40$

Starting with this problem, pupils who demonstrate good fluency, reasoning and problem solving skills are able to use this fact to create others such as:

$5 \times 8 = 40$

$40 \div 5 = 8$

$40 \div 8 = 5$

$8 \times 50 = 400$

$80 \times 50 = 4000$

$8 \times 5 = 20 \times 2$

$(2 \times 4) \times 5 = 10 \times 4$

$16 \times 2.5 = 40$

$40 \times 8 \neq 5$

$5 \times 8 = 8 + 8 + 8 + 8 + 8$

$5 \times 8 = (5 \times 10) - (5 \times 2)$

$0.8 \times 0.5 = 0.4$

$5 \times 8 = 10 \times 4$

$23\times \sqrt{25} = 40 = 8 \times 5$

$40 = 8 \times 5$

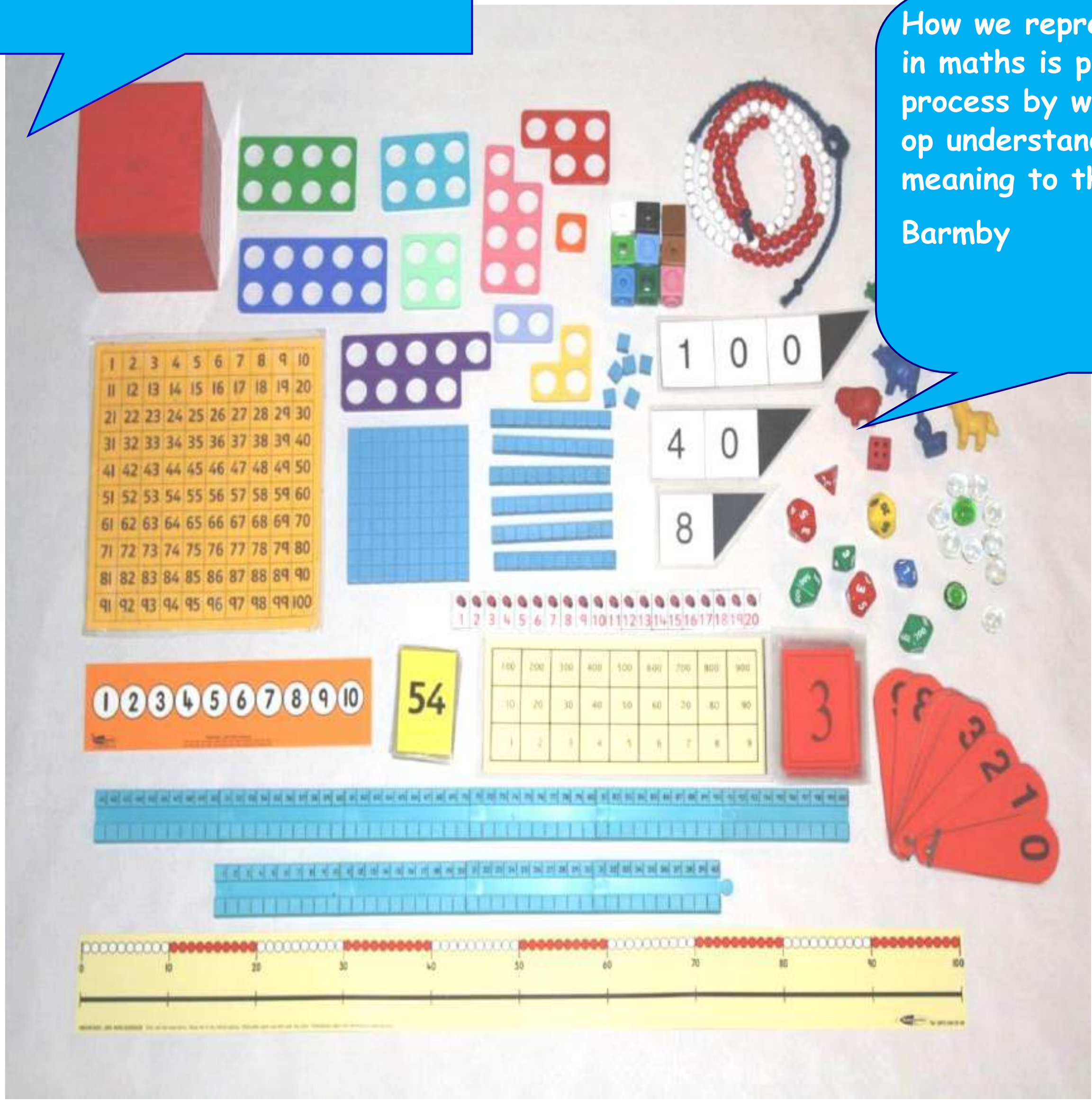
Representations

ST. JOHN BOSCO

CARING FOR EACH OTHER

External representations permit us to talk about mathematical relations and meaning.

Kapult

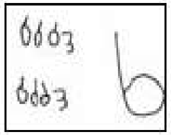
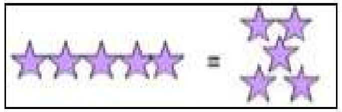


How we represent an idea in maths is part of the key process by which we develop understanding and give meaning to that idea.

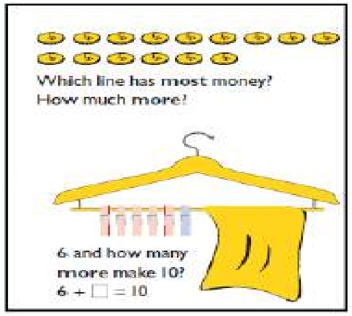
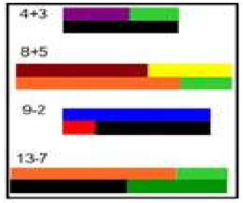
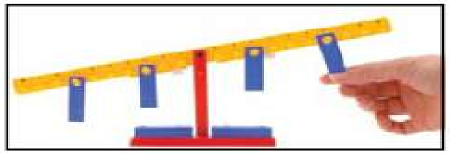
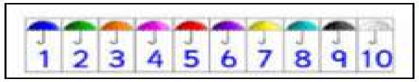
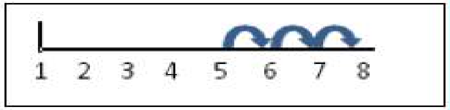
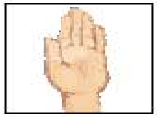
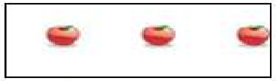
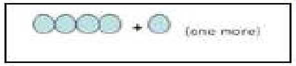
Barmby

At St John Bosco, we use a variety of concrete, pictorial and abstract representations for numbers and calculations. Pupils should have an opportunity to manipulate and experience a variety of models, images, and resources to enable them to choose the most suitable representation for each calculation. In the picture you can see some examples of models, images and resources: arrow cards, bead strings, counters, dice, dienes, digit cards, multilink, number fans, number lines, number tracks, numicon, place value cards, 100 square, sorting objects etc.

Year 1 Addition

Mental Calculations	•Read, write and interpret mathematical statements using symbols +, -, = •Represent and use number bonds and related addition facts within 20 •Add one digit and two-digit numbers up to 20, including zero. •Solve one-step problems using concrete objects and pictorial representations, and missing number problems such as 7 = ☐ - 9 •Given a number, identify (and use the language) one more
Written Calculations	•Begin to compare (what's the same/different?) for commutative sums e.g 3 +7 = 7 + 3 •*Memorise and reason with number bonds to 10 & 20 in several forms* •Add using objects, Numicon, cubes etc and number lines and tracks •Check with everyday objects •Ensure pre-calculation steps are understood, including: •Counting objects (including solving simple concrete problems) •Conservation of number: •Recognise place value in numbers beyond 20 •Counting as reciting and as enumerating   

Year 1 Addition

Representations to support mental and written calculations.	Use a range of concrete and pictorial representations, including:      Number lines    Bead strings  Number tracks   Real everyday objects 
Links from other strands	• *Combine and increase numbers, counting forwards and backwards.* • *Develop the concept of addition and subtraction and … use these operations flexibly.* • *Discuss and solve problems in familiar practical contexts, including using quantities* • *Compare, describe and solve practical [measure] problems e.g. longer, more than, heavier than* • Problems terminology should include: put together, add, altogether, total, take away, distance between, difference between, more than and less than.

Year 2 Addition

Mental Calculations

Add numbers using concrete objects, pictorial representations, and mentally, including:

- a two-digit number and ones
- a two-digit number and tens
- two two-digit numbers
- adding three one-digit numbers

$17 + 2 = 19$ $\quad 12 + 4 = 16$

$57 + 2 = 59$ $\quad 32 + 34 = 66$

- **Recall and use addition and subtraction facts to 20 fluently, and derive and use related facts up to 20.**

Written Calculations

- Demonstrate the commutative law of addition
- Re-partition numbers eg.
- Use a hundred square
- *Check calculations using inverse and by adding numbers in different order*
- Begin to *record addition in columns to support place value and prepare for formal written methods with larger numbers*

$12 + 30 = 30 + 12$

$\square + 25 = 25 + 41$

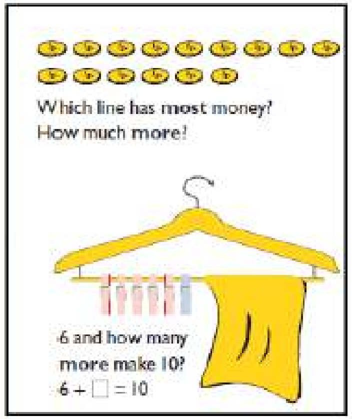
$65 = 60 + 5$
$65 = 50 + 15$
$65 = 40 + 25$
$65 = 30 + 35$
$65 = 20 + 45$
$65 = 10 + 55$

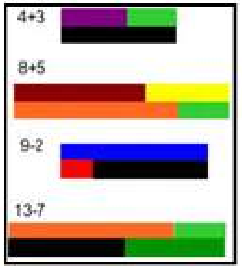
$$\begin{array}{c} 30 + 4 \\ \underline{20 + 5} \\ 50 + 9 \end{array}$$

Year 2 Addition

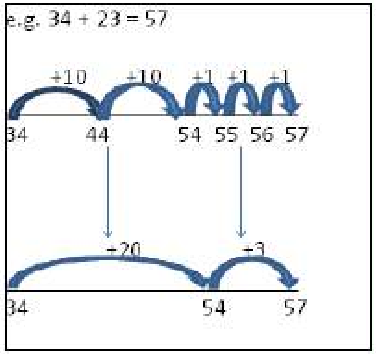
Representations to support mental and written calculations.

Use a range of concrete and pictorial representations, including:

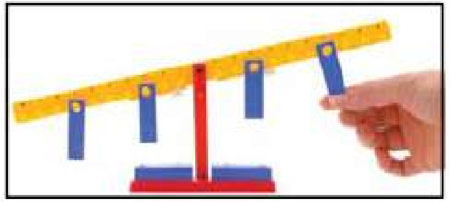


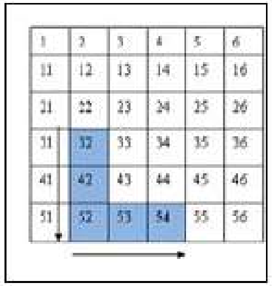


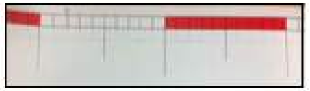
Bead strings



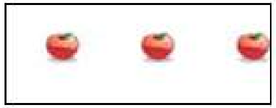
Number lines







Number tracks



Real everyday objects

Fractions

Counting in fractions up to 10, starting from any numbers and using the 1/2 and 2/4 equivalence on the number line

Year 2 Addition

Links from other strands	•Solve problems: •Using concrete objects, pictorial representations (numbers, quantities & measures) •Applying increasing knowledge of mental & written methods •Partition numbers in different ways •*Discuss and solve problems that emphasise the value of each digit in two-digit numbers* (They should) develop the concept of addition and subtraction and ... use these operations flexibly. (*Number-addition and subtraction, Non-statutory guidance.)*

Year 3 Addition

Mental Calculations

Add numbers mentally, including:

- **a three-digit number and ones**
- **a three-digit number and tens**
- **a three digit number and hundreds**

- Partition all numbers and recombine, start with TU + TU then HTU + TU
- Use straws, dienes, place value counters, empty number lines

Common Mental Calculations

Partitioning and recombining, Doubles and near doubles

Use number pairs to 10 and 100, Adding near multiples of 10 and adjusting, Using patterns of similar calculations

Using known number facts

Bridging through ten, hundred

Complementary addition

Written Calculations

Add numbers with up to three digits, using formal written (columnar) methods

Add to three digit numbers using physical and abstract representations (e.g. straws, dienes, place value counters, empty number lines)

- raws, dienes, place value counters, empty number lines

30 + 4	34
20 + 5	+25
50 + 9	59

200 + 30 + 4	234
500 + 20 + 7	+ 527
700 + 60 + 1	761
10	1

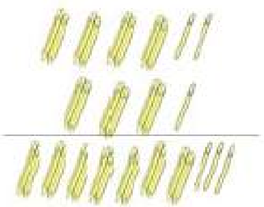
Revert to concrete representations if children find expanded/column methods difficult

Year 3 Addition

Representations to support mental and written calculations

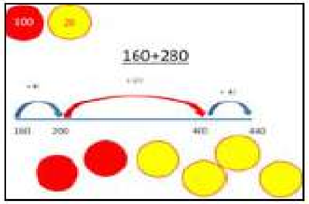
Use a range of concrete, pictorial and abstract representations, including those below

Bundles of straws



42 + 31 = 73

0 + 50 + 3
10 + 40 + 3
20 + 30 + 3
30 + 20 + 3
40 + 10 + 3
50 + 0 + 3



I can explain my method using representations

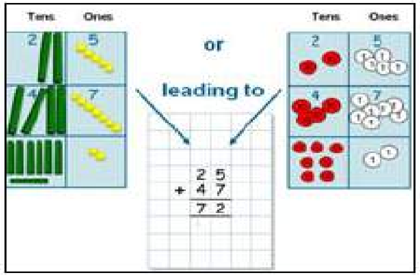
76 + 21

= 70 + 6 + 20 + 1

= 90 + 7 = 97

Partitioning and recombining

What is the same and what is different about all these methods?



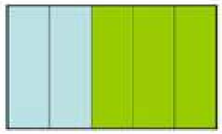
Dienes and place value counters

Fractions

Addition of fractions with the same denominator within one whole.

Addition of fractions with the same denominator

$\frac{2}{5} + \frac{3}{5} = \frac{5}{5}$



Links from other strands

Pupils should estimate the answers to a calculation & use inverse operations to check answers.
Add amounts of money using both £ and p in practical contexts.
Measure, compare and add lengths (m/cm/mm), mass (kg/g) & volume/capacity (l/ml)

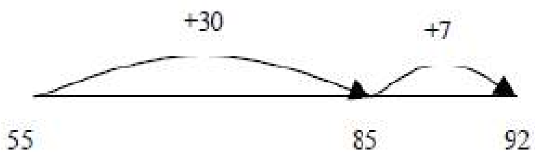
Year 4 Addition

Informal methods to support mental Calculations

Practise mental methods with increasingly large numbers

$$55 + 37 = 55 + 30 + 7$$
$$= 85 + 7$$
$$= 92$$

Consolidate partitioning and re-partitioning
Use compensation for adding too much/little and adjusting
Use straws, Dienes, place value counters, empty number lines etc.



I know that 63 + 29 is the same as 63 + 30 –1

Common Mental Calculations

Partitioning and recombining, Doubles and near doubles

Use number pairs to 10 and 100, Adding near multiples of 10 and adjusting, Using patterns of similar calculations

Using known number facts

Bridging through ten, hundred

Complementary addition

Written Calculations

Add numbers with up to four digits, using the formal written (columnar) method

Add three digit numbers using columnar method and then move onto 4 digits.
Include decimal addition for money

789 + 642 becomes

$$\begin{array}{r} 7\ 8\ 9 \\ +\ 6\ 4\ 2 \\ \hline 1\ 4\ 3\ 1 \\ \hline {}_{1}\ {}_{1}\ \ \end{array}$$

Answer: 1431

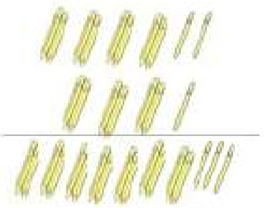
Revert to expanded methods if children find formal calculation method difficult

Year 4 Addition

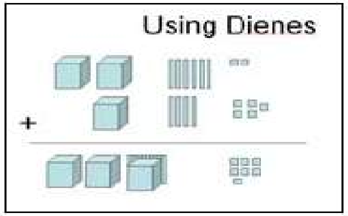
Representations to support mental and written calculations.

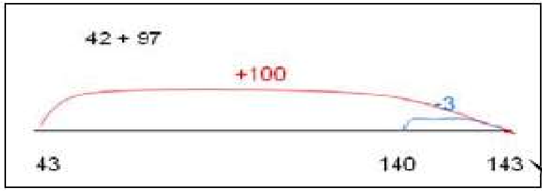
Use physical/pictorial representations alongside expanded and columnar methods.

Bundles of straws



42 + 31 = 73





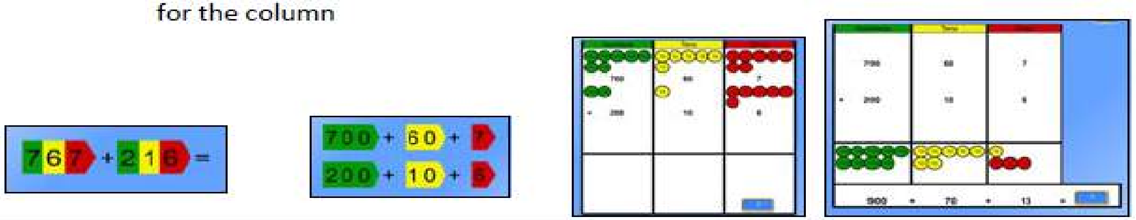
Compensating in mental addition

£12.32
+ £11.81
£24.13
1

0 + 50 + 3
10 + 40 + 3
20 + 30 + 3
30 + 20 + 3
40 + 10 + 3
50 + 0 + 3

Re-partitioning

Place value cards & counters to counters, support the expanded method in readiness for the column

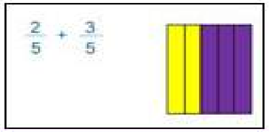


Ask what is the same and what is different about all these methods?

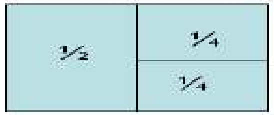
Fractions

Addition of fractions with the same denominator *to become fluent through a variety of increasingly complex problems beyond one whole*

Counting using simple fractions and decimals, both forwards and backwards



$$\frac{1}{2} + \frac{2}{4} = \frac{2}{4} + \frac{2}{4} = 1$$



Links from other strands

- Estimate and use inverse operations to check answers.
- Solve addition and subtraction two step problems in context, deciding which operations and methods to use and why
- Identify, represent and estimate numbers using different representations. (Place value)
- Recognise the place value of each digit in a four-digit number.
- Estimate, compare and calculate different measures, including amounts money in £ and p (including fractions and decimals)

Year 5 Addition

Informal methods to support mental Calculations	
	• Add numbers mentally with increasingly large numbers, e.g. 12 462 + 2300 = 14 762 • Mentally add tenths, and one-digit numbers and tenths • Add decimals, including a mix of whole numbers and decimals, decimals with different numbers of places, and complements of 1 (e.g. 0.83 + 0.17 = 1) Children use representation of choice Refer back to pictorial and physical representations when needed. **Common Mental Calculations** Partitioning and recombining, Doubles and near doubles Use number pairs to 10 and 100, Adding near multiples of 10 and adjusting, Using patterns of similar calculations Using known number facts Bridging through ten, hundred, tenth Complementary addition
Written Calculations	**Add whole numbers with more than four digits, using the formal written (columnar) method** Add three digit numbers using columnar method and then move onto 4 digits. Include decimal addition for money 24172m + 5929m 30101m 1 1 1 1 £563.14 + £207.88 £771.02 1 1 1 **Revert to expanded methods if children find formal calculation method difficult (see Y3)**

Year 5 Addition

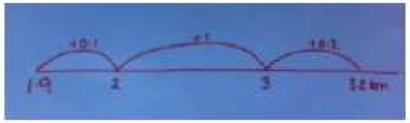
Represent-ations to support mental and written calculations.

Use physical/pictorial representations alongside columnar methods where needed.

12 462 + 2300
= 12 462 + 2000 + 300
= 14 462 + 300
= 14 762

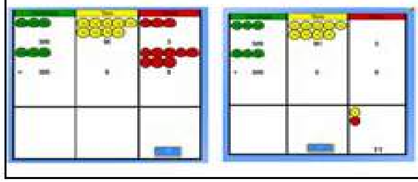
Partitioning and recombining

Ask **what is the same** and **what is different** about all these methods?



Jottings to support mental calculation

Place Value counters to support column addition

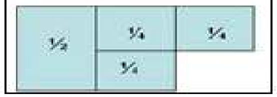


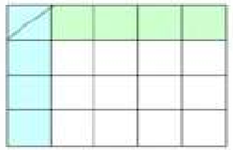
$$\begin{array}{r} 393 \\ +\ 308 \\ \hline 1 \\ \hline 1 \end{array}$$

Fractions

- **Add fractions with the same denominator and denominators that are multiples of the same number** *(to become fluent through a variety of increasingly complex problems and add fractions that exceed 1 as a mixed number)*

$$\frac{1}{2} + \frac{3}{4} = \frac{2}{4} + \frac{3}{4} = \frac{5}{4}$$





$$\frac{1}{4} + \frac{1}{5} = \frac{5}{20} + \frac{4}{20} = \frac{9}{20}$$

Links from other strands

- Solve problems involving up to three decimal numbers.
- Solve addition and subtraction multi step problems in context, deciding which operations and methods to use and why
- Use all four operations to solve problems involving measure [e.g. length, mass, volume, money] using decimal notation,
- Calculate the perimeter of composite rectilinear squares in centimetres and metres
- Use angle sum facts and other properties to make deductions about missing angles
- Solve comparison, sun and difference problems using information presented in a line graph

Year 6 Addition

Informal methods to support mental Calculations

- **Perform mental calculations, including with mixed operations and large numbers** *(more complex calculations)*

Children use representation of choice

Consolidate partitioning and re-partitioning

Use compensation for adding too much/little and adjusting

Refer back to pictorial and physical representations when needed.

Common Mental Calculations

Partitioning and recombining, Doubles and near doubles

Use number pairs to 10 and 100, Adding near multiples of 10 and adjusting, Using patterns of similar calculations, Using known number facts, Bridging through ten, hundred, tenth, Complementary addition

Written Calculations

Add larger numbers using the formal written (columnar) method

Add three digit numbers using columnar method and then move onto 4 digits.

Include decimal addition for money

```
  £563.14
+ £207.88
  £771.02
    1 1 1
```

789 + 642 becomes

```
    7 8 9
+   6 4 2
  1 4 3 1
    1 1
```

Answer: 1431

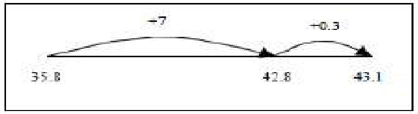
Revert to expanded methods if children find formal calculation method difficult (see Y3)

Representations to support mental and written calculations.

Use physical/pictorial representations alongside columnar methods where needed. Ask what is the same and what is different?

12 462 + 2300
= 12 462 + 2000 + 300
= 14 462 + 300
= 14 762

Partitioning and recombining

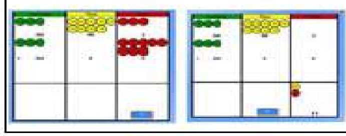


234 kg + 49 kg = 273 kg

```
200 + 30 + 4
      40 + 9
200 + 70 + 13
```

I can explain my method using place value counters

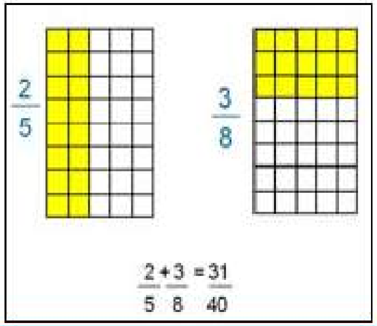
Place Value counters to support column addition



```
  393
+ 308
    1
    1
```

What is the same and what is different about all these methods?

Year 6 Addition

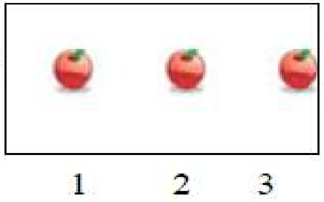
Fractions	• **Add fractions with different denominators and mixed numbers, using the concept of equivalent fractions** • Start with fractions where the denominator of one fraction is a multiple of the other (e.g. 1/2 + 1/8 = 5/8) and progress to varied and increasingly complex problems • Practise calculations with simple fractions and decimal equivalents to aid fluency 
Links from other strands	• Use their knowledge of the order of operations to carry out calculations involving the four operations (BIDMAS) • Solve problems involving all four operations • Algebra: use symbols and letters to represent variable and unknowns *e.g. a + b = b + a* • Solve problems involving the calculation and conversions of units of measure, using decimal notation of up to three decimal places where appropriate • *Using the number line, pupils use, add and subtract positive and negative integers for measures such as temperature* • Calculate and interpret the mean as an average • Interpret and construct pie charts and line graphs and use these to solve problems • Find missing angles, *and express geometry relationships algebraically (e.g. d=2xr)*

Year 1 Subtraction

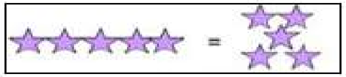
Mental Calculations

Subtract one digit and two-digit numbers to 20, including zero.
Read, write and interpret mathematical statements using symbols (+, -, =) signs.
Represent and use number bonds and related addition facts within 20
Solve one-step problems using concrete objects and pictorial representations, and missing number problems such as 7 = - 9

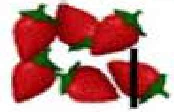
Memorise and reason with number bonds
Add using objects, Numicon, cubes etc and number lines and tracks
Check with everyday objects
Ensure pre-calculation steps are understood, including:
Counting objects,



Conservation of number



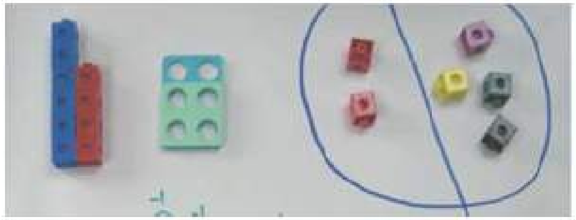
Understand subtraction as 'take away'



Find a 'difference' by counting up:

Written Calculations

Subtract one-digit and two-digit numbers to 20, including zero.

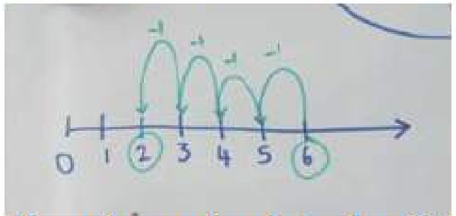


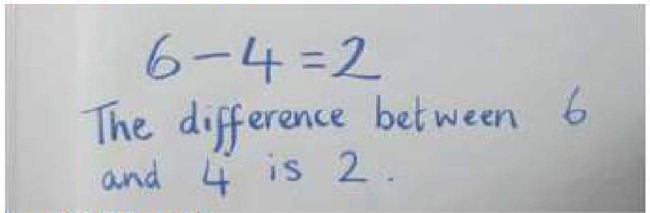
7 – 3 = □, 7 - □ = 4

□ - 3 = 4, 17 – 13 = □

17 - □ = 4

Read, write and interpret mathematical statements involving addition (+), subtraction (–) and equals (=) signs .



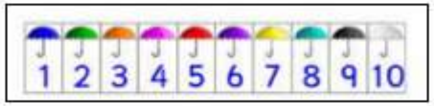


Represent and use number bonds and related subtraction facts within 20.

Year 1 Subtraction

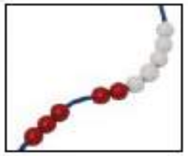
Representations to support mental and written calculations

Use a range of concrete and pictorial representations, including:

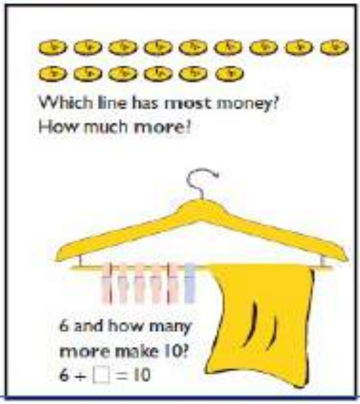


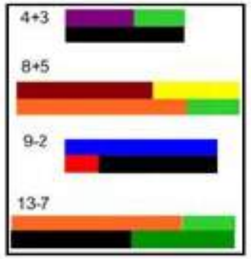
Straw bundles

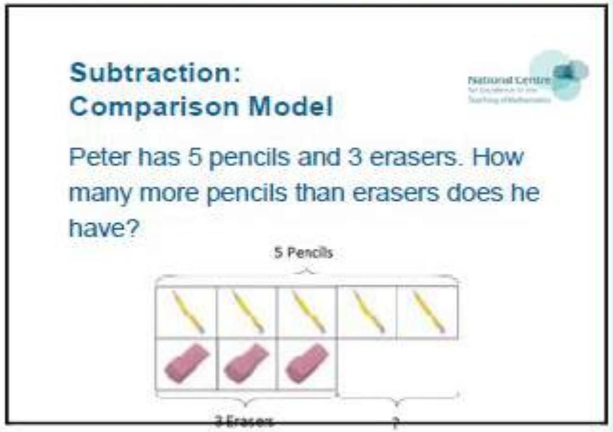
Hands, and children themselves.



Bead strings, number tracks and lines







Links from other strands

Pupils should combine and increase numbers, counting forwards and backwards.

(They should) develop the concept of addition and subtraction and … use these operations flexibly.

Problems should include the terms: put together, add, altogether, total, take away, distance between, difference between, more than and less than, so that pupils develop the concept of addition and subtraction and are enabled to use these operations flexibly.

(Number-addition and subtraction, Non-statutory guidance.)

Pupils discuss and solve problems in familiar practical contexts . *(Non-statutory guidance.)*

Pupils compare, describe and solve practical (measurement) problems .

(Measurement)

Year 2 Subtraction

Mental Calculations

Add and subtract numbers using concrete objects, pictorial representations, and mentally, including:

- a two-digit number and ones
- a two-digit number and tens
- two two-digit numbers
- adding three one-digit numbers

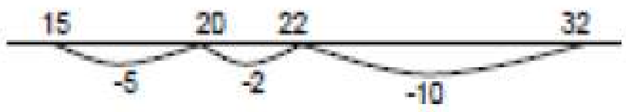
Jottings to support informal methods:

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

54—32 = 22

Bridge through 10 where necessary

32 - 17



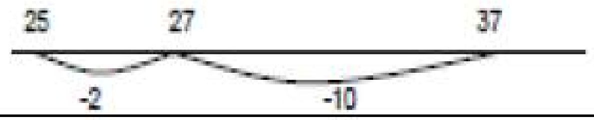
Written Calculations

Written recording:

37 – 12 = 37 – 10 - 2

= 27 – 2

= 25



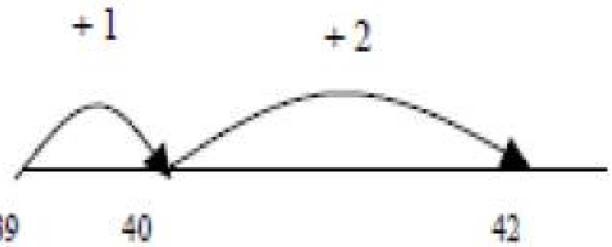
<u>- = signs and missing numbers</u>

Continue using a range of equations as in Year 1 but with appropriate numbers.

Extend to 14 + 5 = 20 - □

<u>Find a small difference by counting up</u>

42 – 39 = 3



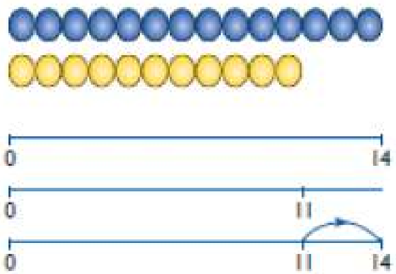
Year 2 Subtraction

Representations to support mental and written calculations.

<u>Informal methods to support written subtraction calculations</u>

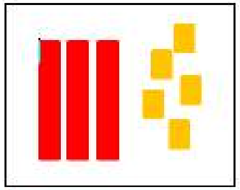
Practical portioning of a 2-digit number

In Year 1 leads to:

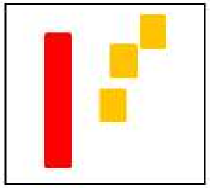


The difference between 11 and 14 is 3.
14 – 11 = 3
11 + ☐ = 14

Bundles of straws or dienes to represent and partition 2 digit numbers.
Subtract (without decomposition) using partitioning and equipment, e.g.



To calculate 35-22, remove 22.



Then record: **35-22=13.**

Continue to use of a range of concrete and pictorial representations from Year 1—including Bar model to support understanding of **difference. (See below.)**

Fractions

Pupils should count in fractions up to 10, starting from any number and using the equivalence on the number line (for example, 1 ¼, 1 ½, 1 ¾, 2.)

Use concrete and pictorial models of fractions to assist with counting e.g. paper cups, plates, shapes etc.

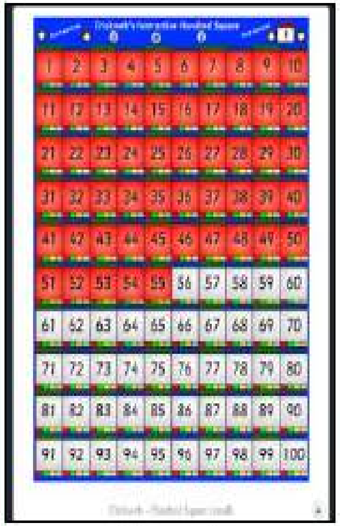
Year 2 Subtraction

Links from other strands

Recall and use addition and subtraction facts to 20 fluently,

and derive and use related facts up to 100.

Pupils should partition numbers in different ways (for example, 23 = 20 + 3 and 23 = 10 + 13) to support subtraction.



55 + 45 = 100
45 + 55 = 100
35 + 65 = 100
100—55 = 45
100—45 = 55
100—35 = 65

Solve problems with addition and subtraction:

- **using concrete objects and pictorial representations, including those involving numbers, quantities and measures**
- **applying their increasing knowledge of mental and written methods**
- ***Pupils extend their understanding of the language of addition and subtraction to include sum and difference.***

Year 3 Subtraction

Mental Calculations

Add and subtract numbers mentally, including:
***a three-digit number and ones**
***a three-digit number and tens**
***a three-digit number and hundreds.**

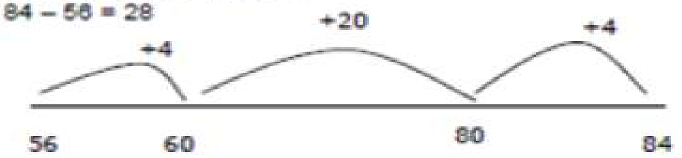
Use a number line, dienes, hundred squares, two-hundred squares, and similar representations, to support mental calculations. (See Representations section below.)

101	102	103	104	105	106	107	108	109	110
111	112	113	114	115	116	117	118	119	120
121	122	123	124	125	126	127	128	129	130
131	132	133	134	135	136	137	138	139	140
141	142	143	144	145	146	147	148	149	150
151	152	153	154	155	156	157	158	159	160
161	162	163	164	165	166	167	168	169	170
171	172	173	174	175	176	177	178	179	180
181	182	183	184	185	186	187	188	189	190
191	192	193	194	195	196	197	198	199	200

Use known number facts and place value to subtract
Continue as in Year 2 but with appropriate numbers e.g.
97 – 15 = 72

With practice, children will need to record less information and decide whether to count back or forward. It is useful to ask children whether counting up or back is the more efficient for calculations
such as 57 – 12, 86 – 77 or 43 – 28.

Pencil and paper procedures
Complementary addition
84 – 56 = 28



Written Calculations

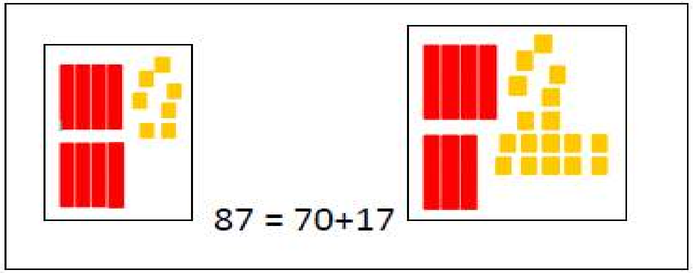
Add and subtract numbers with up to three digits, using formal written methods of columnar addition and subtraction.

(1)Extended columnar - no exchange

Extended method 87 - 53 =

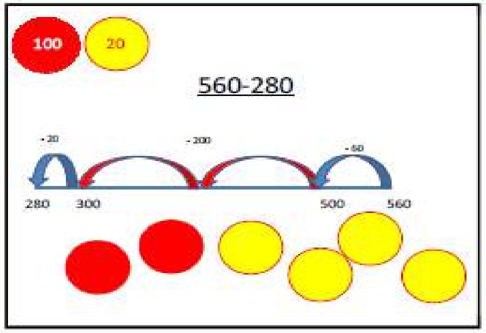
80 and 7
- 50 and 3
30 and 4 = 34

(2) Extended columnar – with exchange:
87-58 becomes
70 +1 7
-50 + 8
20 + 9

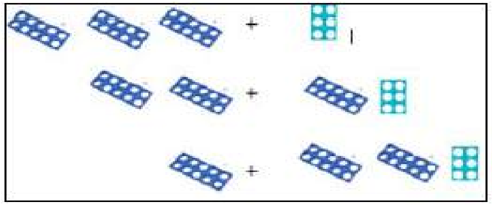


Year 3 Subtraction

Representations to support mental and written calculations.



Partitioning and re- partitioning support the understanding of place-value.



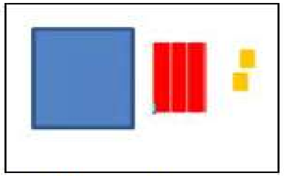
$30 + 6$

$20 + 16$

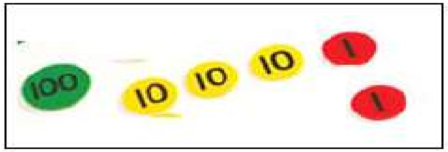
$10 + 26$

All of these representations still comprise the amount of 36.

Introduce transition from concrete place value representations, (e.g. dienes or straws), to pictorial – such as place value counters or money.



132 in dienes



132 in place value counters.

Revert to concrete manipulatives and expanded methods whenever difficulties arise

Fractions

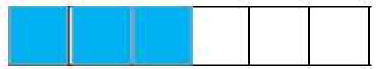
Count up and down in tenths.

Add and subtract fractions with the same denominator within one whole.

$$\frac{1}{6} + \frac{1}{6} + \frac{1}{6} = \frac{3}{6} = \frac{1}{2}$$

Adding Fractions

Bar model



Links from other strands

Money and calculating duration of events (with number lines.)

For example: *"Add and subtract amounts of money to give change, using both £ and p in practical contexts."*

"Compare durations of events [for example to calculate the time taken by particular events or tasks]." (Measurement)

Year 4 Subtraction

Mental Calculations

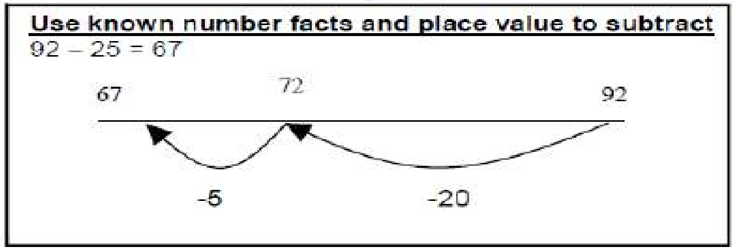
Continue to practise mental methods with increasingly large numbers to aid fluency. *(From Non– Statutory Guidance).*

Methods to support fluent calculation and encourage efficiency of method:

- Find a small difference by counting up.
  E.g. 5003—4996
- Subtract nearest multiple of ten and adjust.
- Partition larger numbers

Whenever possible, children should be encouraged to visualise number lines and other basic, supporting representations to promote fluent work without jottings.

This could be done using an empty number line. Children should recall and use number facts to reduce the number of steps.



Written Calculations

Add and subtract numbers with up to 4 digits using the formal written methods of columnar addition and subtraction where appropriate.

Build on formal, extended method (*See Year 3)* using exchange wherever necessary.

Continue to use representations and manipulatives to develop understanding of place value.

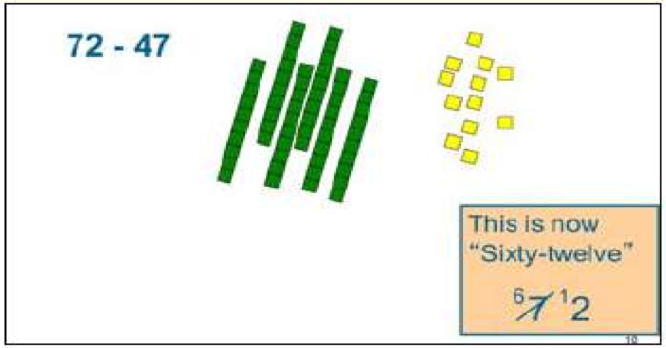
372—147 =

$$\begin{array}{r} 300 + 70 + 2 \\ \underline{-100 + 40 + 7} \end{array} \rightarrow \begin{array}{r} 300 + 60 + 12 \\ \underline{-100 + 40 + \ \ 7} \\ 200 + 20 + \ \ 5 \end{array} \rightarrow \begin{array}{r} \qquad 60 \qquad \ \ 1 \\ 300 + \cancel{70} + \ \ 2 \\ \underline{-\ \ 100 + 40 + \ \ 7} \\ 200 + 20 + \ \ 5 \end{array}$$

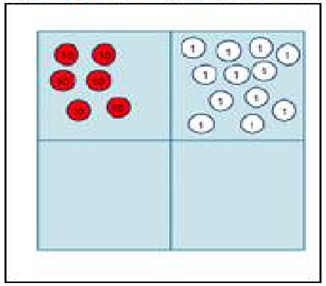
Apply understanding of subtraction with larger integers to that of decimals in context of money and measures. (See Year 5.)

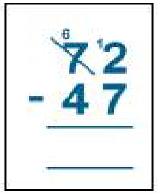
Year 4 Subtraction

Representations to support mental and written calculations.



Dienes blocks or place value counters can be used to model calcu-lations and the under-lying place value concepts.

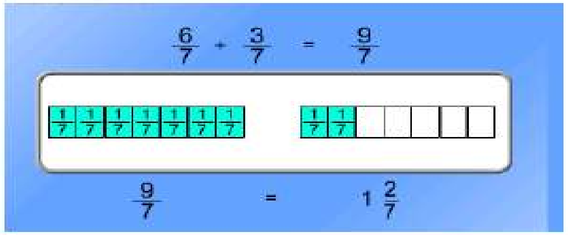




Use physical and / or pictorial representations and expanded algorithms alongside columnar methods. Ask: *What is the same? What's different?*
Compare and discuss the suitability of different methods in context.
Pupils **decide which operations and methods to use and why.**

I would count on using a number line to calculate 5003-4896; because the numbers are close together.

Fractions



Count up and down in hundredths.
Add and subtract fractions with the same denominator .
Solve simple measure and money problems involving fractions and decimals to two decimal places.

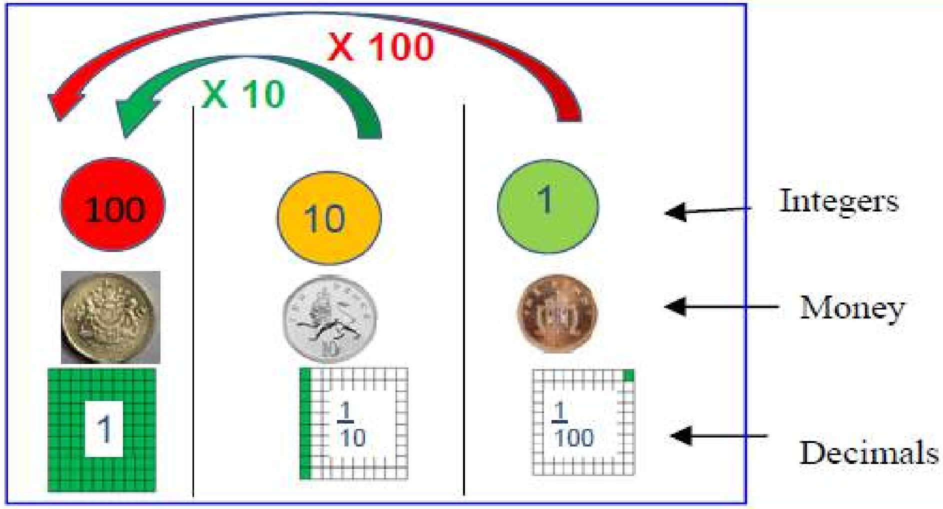
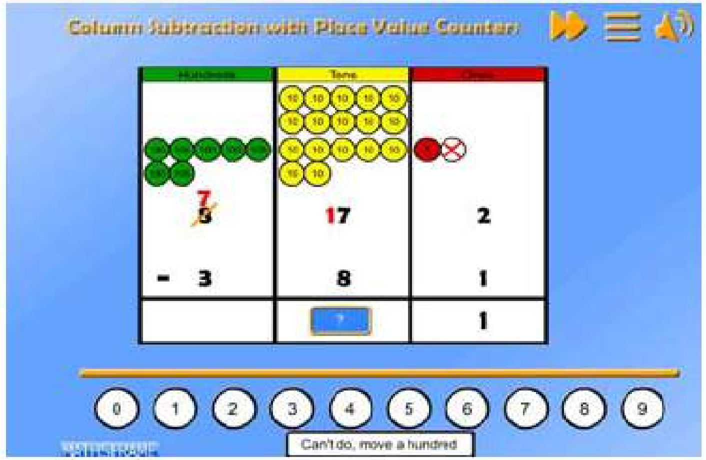
Links from other strands

Identify, represent and estimate numbers using different representations. (*Place value*)
Recognise the place value of each digit in a four-digit number.
Estimate and use inverse operations to check answers to a calculation .
Solve addition and subtraction two-step problems in contexts, deciding which operations and methods to use and why.
Estimate, compare and calculate different measures, including money in pounds and pence.

Year 5 Subtraction

Mental Calculations	• **Subtract numbers mentally with increasingly large numbers.** **E.g. 12 462 – 2300 = 10 162** • **Use rounding to check answers to calculations and determine, in the context of a problem, levels of accuracy .** • ***Pupils practise adding and subtracting decimals, including a mix of whole numbers and decimals, decimals with different numbers of decimal places, and complements of 1 (for example, 1 - 0.17 = 0.83).*** • ***Pupils mentally add and subtract tenths, and one-digit whole numbers and tenths.***	Basic Mental Strategies for Subtraction ◆ Find differences by counting up ◆ Partitioning ◆ Applying known facts ◆ Bridging through 10 and multiples of 10 ◆ Subtracting 9, 11 etc. by compensating ◆ Counting on to, or back from the largest number *National Curriculum 1999* *Which method works best? Why? How else could we do it?* Children use, or visualise, representation of choice. Refer back to physical representations as required.
Written Calculations	**Add and subtract whole numbers with more than 4 digits, including using formal written methods (columnar addition and subtraction).** ***(Pupils) practise adding and subtracting decimals.*** Begin with three-digit numbers using formal, columnar method; then move into four-digit numbers. As in Year 4, compare physical and / or pictorial representations and expanded algorithms alongside columnar methods. Ask: *What is the same? What's different?* Compare and discuss the suitability of different methods, (mental or written), in context. Revert to expanded methods whenever difficulties arise £17.34—£12.16 *What is the same about these models? What's different?* 1000+700+20+14p - 1000+200+10+ 6p 500+10+ 8p → 2 17~~3~~4p - 1216p 518p → £ 2 17.~~3~~4 - 12.16 5.18 Relate place value of decimals with that of whole numbers using representations. See below.	

Year 5 Subtraction

Representations to support mental and written calculations.	  Use physical and pictorial representations to stress the place value relationships between money, decimals and whole numbers. A place value mat such as the this one could be used, moving away from the traditional: *Hundreds, tens and ones* model used in Lower KS2 and KS1.
Fractions	**Subtract fractions with the same denominator and denominators that are multiples of the same number.** **(*Include fractions exceeding 1 as a mixed number.)*** **Solve problems involving number up to three decimal places .** **They mentally add and subtract tenths, and one-digit whole numbers and tenths.**
Links from other strands	**Solve problems involving addition, subtraction, multiplication and division and a combination of these, including understanding the meaning of the equals sign.** **Use all four operations to solve problems involving time, money and measure using decimal notation.; (up to 3d.p.)**

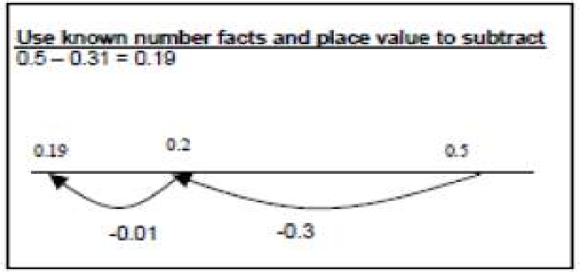
Year 6 Subtraction

Mental Calculations

Children:

- **Perform mental calculations, including with mixed operations and large numbers.**
- **Use estimation to check answers to calculations and determine, in the context of a problem, an appropriate degree of accuracy.**
- ***They undertake mental calculations with increasingly large numbers and more complex calculations.***

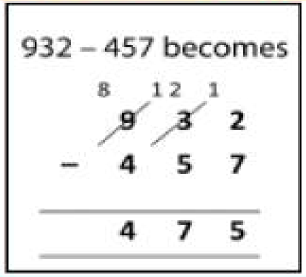
Children draw on basic, Mental subtraction Strategies, (See Year 5.)
Children use, or visualise, representation of choice.
Refer back to physical representations as required.



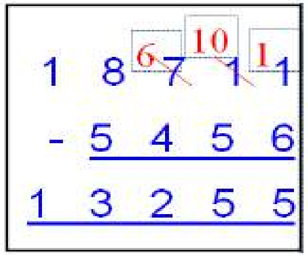
Written Calculations

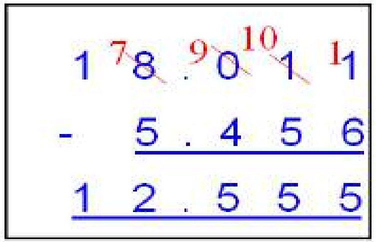
Add and subtract whole numbers with more than 4 digits, including using formal written methods (columnar addition and subtraction). Solve problems involving the calculation and conversions of units of measure, using decimal notation of up to three decimal places where appropriate. (MEASURES)

Move towards consolidation of formal, columnar method.
For more complex calculations, with increasingly larger or smaller numbers, compare representations and expanded algorithms alongside columnar methods. Ask: What is the same? What's different?
Compare and discuss the suitability of different methods, (mental or written), in context.
Revert to expanded methods whenever difficulties arise

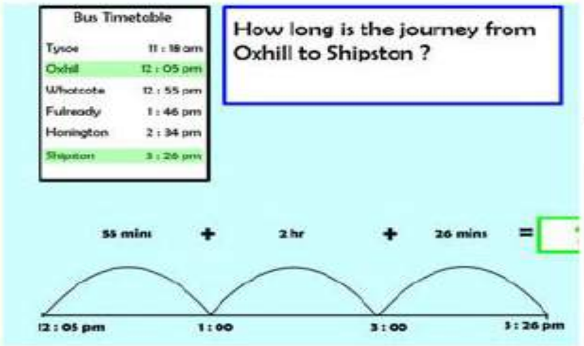
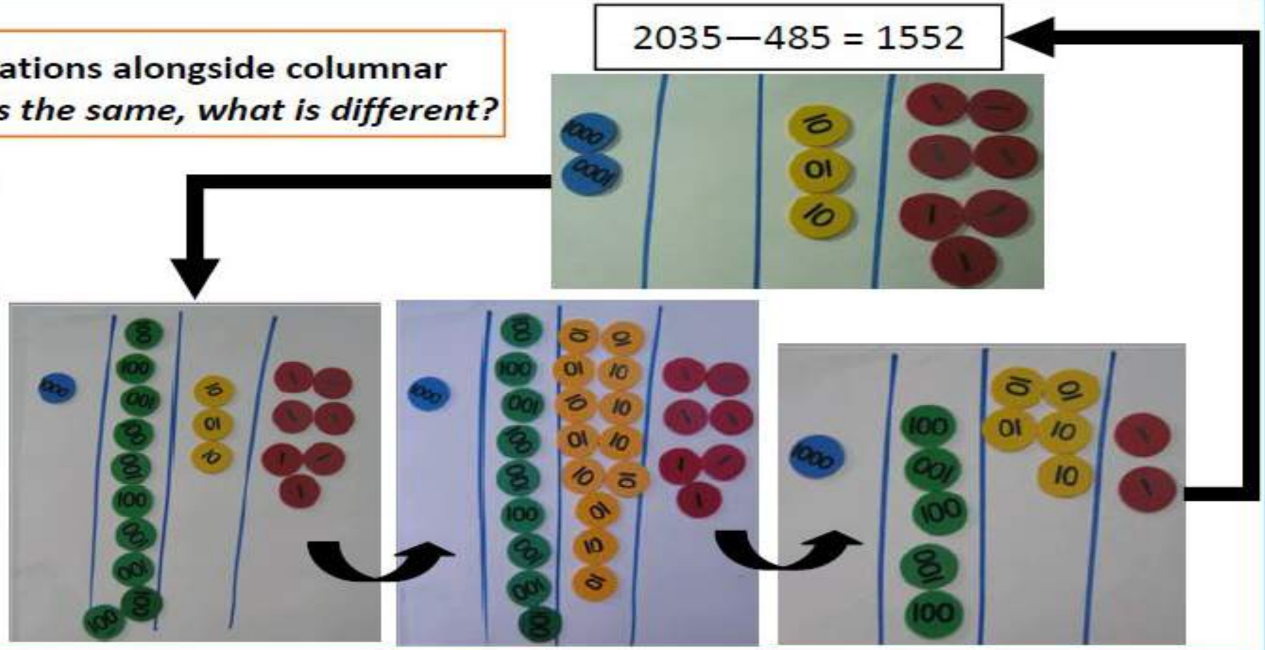


Consolidate columnar methods , paying particular attention to the occurrence of zeros as place holders.





Year 6 Subtraction

Representations to support mental and written calculations.	Use physical/pictorial representations alongside columnar methods where needed. *What is the same, what is different?*  

Fractions	**Add and subtract fractions with different denominators and mixed numbers.** ***They practise calculations with simple fractions and decimal fraction equivalents to aid fluency.***
Links from other strands	**Use their knowledge of the order of operations to carry out calculations involving the four operations (BIDMAS)** **Solve problems involving all four operations** **Algebra: use symbols and letters to represent variable and unknowns e.g. a + b = b + a** **Using the number line, pupils use, add and subtract positive and negative integers for measures such as temperature.**

Year 1 Multiplication

Mental Calculations

- **solve one-step problems involving multiplication and division, by calculating the answer using concrete objects, pictorial representations and arrays with the support of the teacher.**
- **Count in multiples of twos, fives and tens** with equipment, songs & rhythms, and including by rote
  - Counting 2s e.g. counting socks, shoes, animal legs…
  - Counting in 5 s e.g. counting fingers, fingers in gloves, toes …
  - Counting in 10s e.g. counting fingers, toes …
- Doubles up to 10
- Recognising odd and even numbers
- Write as a number pattern (e.g. 5, 10, 15…; 2, 4, 6…; 10, 20, 30…)

What's the sequence?

What comes next?

Written Calculations

It is important to use a range of models to develop understanding of multiplication, and that children make connections between arrays, number patterns, and counting in twos, fives and tens

Although there is no statutory requirement for written multiplication in Year 1, it may be helpful to encourage children to begin to write it as a repeated addition sentence in preparation for Year 2

E.g. 2 + 2 + 2 + 2 = 8

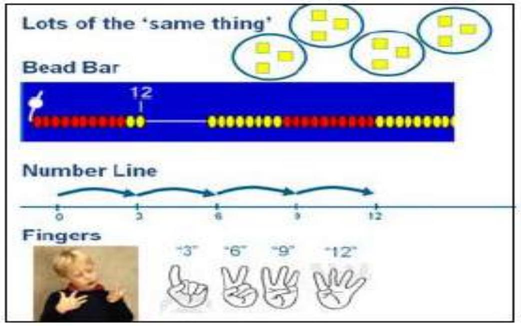
Year 1 Multiplication

Representations to support mental and written calculations.

Use a range of concrete and pictorial representations, including:

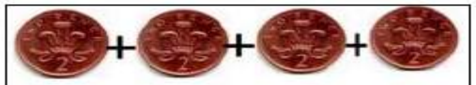
There are 3 sweets in one bag.
How many sweets are there in 5 bags?

4 groups of 3
3 groups of 4



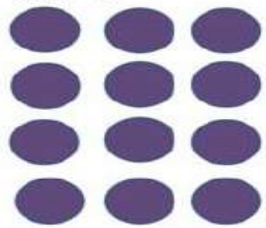
2 groups of 5 (5 x 2) using Numicon

"2 strawberries 3 times"
2 x 3 = 6
2 + 2 + 2 = 6

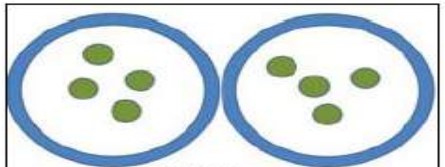


4 groups of 2p
2p multiplied by 4
2p x 4 = 8p

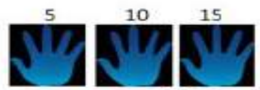
3 + 3 + 3 + 3 = 12
3 multiplied by 4 is 12
3 x 4 = 12



4 x 3 = 12
"4 cakes, 3 times"
4 multiplied by 3



Double 4 in hoops



Links from other strands

- **Count in multiples of twos, fives and tens** (from Number and place value), as above
- *Counting in twos, five and tens from different multiples to develop their recognition of patterns in the number system*
- *They discuss and solve problems in familiar practical contexts, including using quantities.*

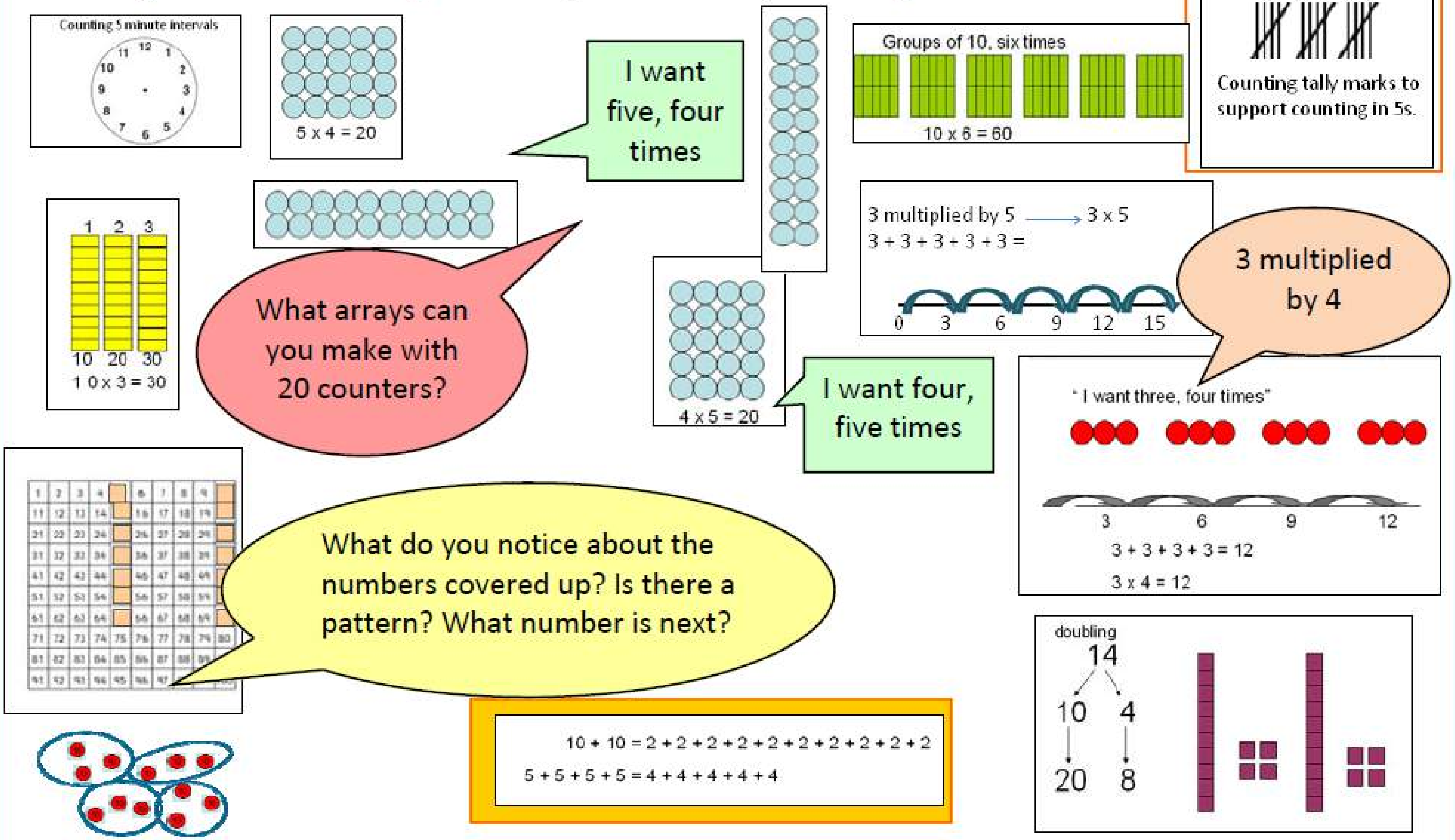
Year 2 Multiplication

Mental Calculations	• **Recall and use multiplication and division facts for the 2, 5 and 10 multiplication tables,** *connecting the 2, 5 and 10 multiplication tables to each other* • *Connect the 10 multiplication table to place value* • **Recognise odd and even numbers** • **show that multiplication of two numbers can be done in any order (commutative)** • *Use a variety of language to describe multiplication and division* • Apply doubling of numbers up to ten to doubling larger numbers I know that the multiples of 2/5/10 are always/never ....
Written Calculations	• **calculate mathematical statements for multiplication and division within the multiplication tables and write them using the multiplication (×), division (÷) and equals (=) signs** • *Begin to use other multiplication tables and recall facts to perform written calculations* • *Use a range of materials and contexts ... including arrays and repeated addition* $7 \times 2 = \square$ $7 \times \square = 14$ $\square \times 2 = 14$ $\triangle \times \square = 14$
Fractions	• **write simple fractions for example, 1/2 of 6 = 3 and recognise the equivalence of 2/4 and 1 /2** • *Begin to relate multiplication and division models to fractions and measures*
Links from other strands	• **solve problems involving multiplication and division, using materials, arrays, repeated addition, mental methods, and multiplication and division facts, including problems in contexts.** • *Use commutativity and inverse relations to develop multiplicative reasoning (e.g. 4 x 5 = 20 and 20 ÷ 5 = 4)* • **Statistics—interpret and consttruct simple pictograms, tally charts and block diagrams** • Measurement— counting 5 minute intervals on a clock face • Place value **count in steps of 2, 3 and 5 from 0 and in tens from any number, forwards and backwards**

Year 2 Multiplication

Representations to support mental and written calculations.

Use a range of concrete and pictorial representations, including:



Year 3 Multiplication

Mental Calculations

- **recall and use multiplication and division facts for the 3, 4 and 8 multiplication tables** (and 2, 5 and 10 multiplication tables from Y2)
- Use doubling to connect 2, 4 and 8 multiplication tables
- Develop efficient mental methods using commutativity and associativity
- Derive related multiplication and division facts
- **calculate mathematical statements for multiplication using the multiplication tables that they know, including for two-digit numbers times one-digit numbers, using mental methods**
- Partitioning: multiply the tens first and then multiply the units, e.g. $57 \times 6 = (50 \times 6) + (7 \times 6) = 300 + 42 = 342$
- Children can apply these skills to solve spoken word problems too,
- Include missing number statements e.g $72 \div \square = 8$

The associative law:
$4 \times 12 \times 5 = 4 \times 5\ 12$
$= 20 \times 12$
$= 240$

The commutative law:
$4 \times 12 = 12 \times 4$

I have 8 packets, each containing 12 crayons. How many crayons do I have in total?'

Ensure opportunities to learn multiplication tables through use of visual models, images and also rote learning.

Multiplication and division facts:
$8 \times 4 = 32$, $4 \times 8 = 32$, $32 \div 4 = 8$, $32 \div 8 = 4$

Deriving related facts:
$3 \times 2 = 60$, $6 \div 3 = 2$, $6 \div 2 = 3$
➡ $30 \times 2 = 60$, $60 \div 3 = 20$, $20 = 60 \div 3$

Written Calculations

- **write and calculate mathematical statements for multiplication using the multiplication tables that they know, including for two-digit numbers times one-digit numbers, progressing to formal written methods**
- Estimate before calculating
- Ensure written methods build on/relate to mental methods

Towards the column method …

x	20	4
6	120	24

$120 + 24 = 144$

$$\begin{array}{r} 24 \\ \times\ 6 \\ \hline 120 \\ 24 \\ \hline 144 \end{array}$$

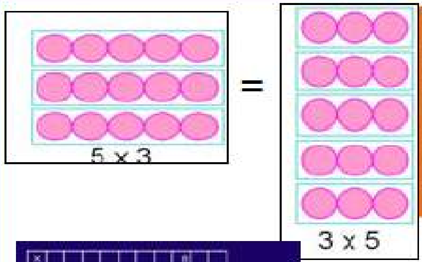
24 × 6 becomes

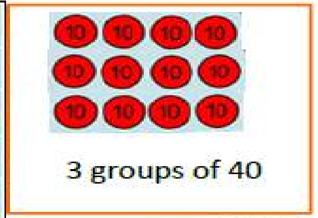
$$\begin{array}{r} 2\ 4 \\ \times\ \ \ 6 \\ \hline 1\ 4\ 4 \\ {}_{2} \\ \hline \end{array}$$

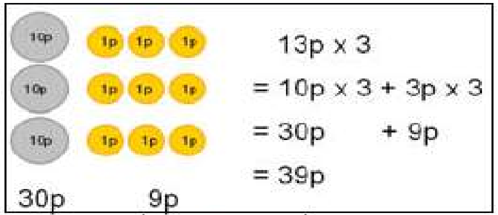
Answer: 144

Year 3 Multiplication

Representations to support mental and written calculations.



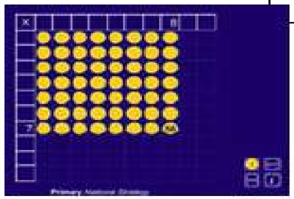


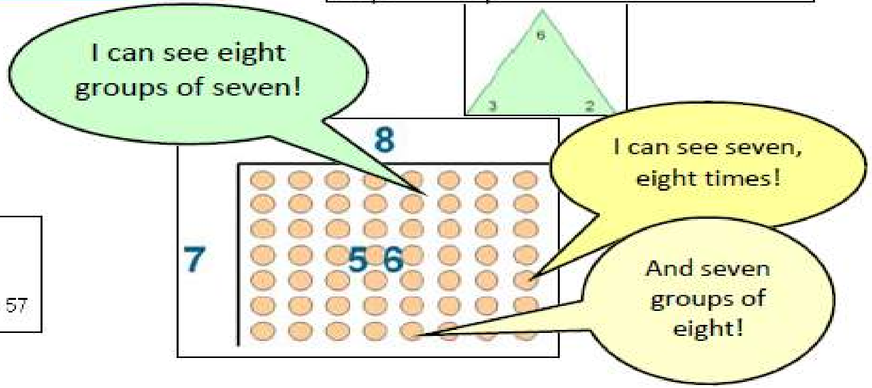


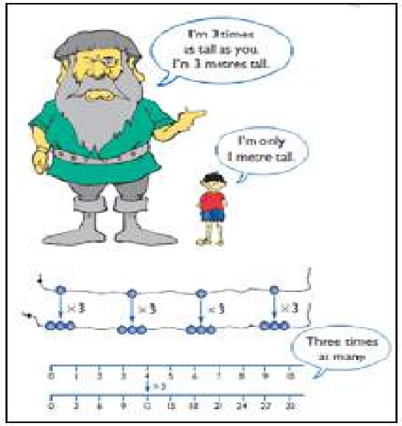
2 digit x 1 digit number:
e.g. 7 x 38 = 266

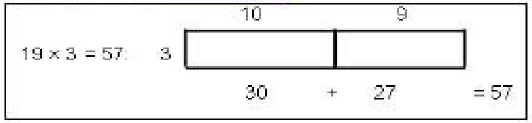
x	30	8
7	210	56

210 + 56 = 266







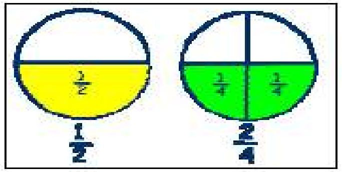


Use arrays for partitioning too

Fractions

- **recognise and show, using diagrams, equivalent fractions with small denominators**

1	2	3	4	5	6	7	8	9	10
2	4	6	8	10	12	14	16	18	20
3	6	9	12	15	18	21	24	27	30
4	8	12	16	20	24	28	32	36	40
5	10	15	20	25	30	35	40	45	50



Links from other strands

- **solve problems, including missing number problems, involving multiplication , including positive integer scaling problems and correspondence problems in which n objects are connected to m objects.**
- *The comparison of measures includes simple scaling by integers (for example, a given quantity or measure is twice as long or five times as high)*
- *Pupils now use multiples of 2, 3, 4, 5, 8, 10, 50 and 100.*
- Pupils understand and use simple scales (for example, 2, 5, 10 units per cm) in pictograms and bar charts with increasing accuracy.

Year 4 Multiplication

Informal methods to support mental Calculations

- **recall multiplication and division facts for multiplication tables up to 12 × 12**
- **use place value, known and derived facts to multiply and divide mentally, including:**
  - **multiplying by 0 and 1;**
  - **dividing by 1;**
  - **multiplying together three numbers**
- **recognise and use factor pairs and commutativity in mental calculations**
- practise mental methods and extend this to three-digit numbers to derive facts, (for example 600 ÷ 3 = 200 can be derived from 2 x 3 = 6)

Using the **distributive** law:
39 x 7 = 30 x 7 + 9 x 7
Using the **associative** law:
(2 x 3) x 4 = 2 x (3 x 4)

Using facts and rules:
2 x 6 x 5 = 10 x 6 = 60

Written Calculations

- **multiply two-digit and three-digit numbers by a one-digit number using formal written layout**
- Estimate before calculating
- Ensure written methods build on/relate to mental methods (e.g. grid method)
- Introduce alongside grid and expanded column methods

	50	4
4	200	16

→

```
   54
 x  4
   16  (4 x 4)
  200  (50 x 4)
  216
```

→

```
  54
 X 4
 216
 1
```

Key skills to support:
- know or quickly recall multiplication facts up to 12 × 12
- understand the effect of multiplying numbers by 10, 100 or 1000
- multiply multiples of 10, for example, 20 × 40;
- approximate, e.g. recognise that 72 × 38 is approximately 70 × 40 = 2800 and use this information to check whether their answer appears sensible

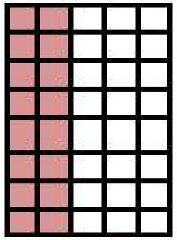
Revert to expanded methods if children find formal calculation method difficult

Fractions

- **recognise and show, using diagrams, families of common equivalent fractions**
- understand the relation between non-unit fractions and multiplication and division of quantities, with particular emphasis on tenths and hundredths.
- make connections between fractions of a length, of a shape and as a representation of one whole or set of quantities.
- use factors and multiples to recognise equivalent fractions and simplify where appropriate

$\frac{4}{10}$ $\frac{6}{15}$ $\frac{8}{20}$ $\frac{10}{25}$ $\frac{12}{30}$ $\frac{14}{35}$ $\frac{16}{40}$

$\frac{2}{5} = \frac{16}{40}$

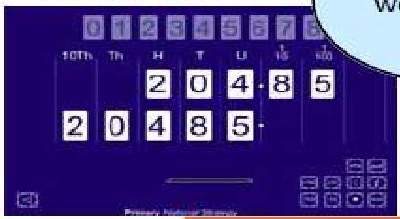


Year 4 Multiplication

Representations to support mental and written calculations.

Ensure children can confidently multiply & divide by 10 and 100, that multiplying by 10 makes the number bigger and all digits move one place to the left, while dividing by 10 makes the number smaller and all the digits move one place to the right.

Moving digits ITP

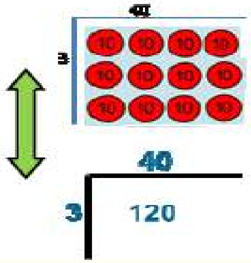


This digit is worth 200

This digit is worth 30

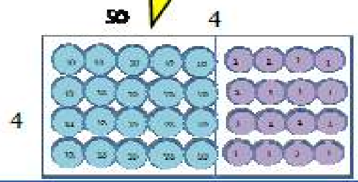
$$\begin{array}{r} 245 \\ \times \quad 6 \\ \hline 1470 \\ {\scriptstyle 2 \quad 3} \end{array}$$

Use arrays made with place value counters to demonstrate the link between multiplication and division. This will support understanding of the grid method.



Children need to understand and apply the language of multiples and factors and use it in solving multiplication and division problems, for example, 'All factors of 36 are multiples of 2, true or false? Find me two factors of 48 that are also multiples of 3.'

I can use place value counters to model the grid method



Links from other strands

- **solve problems involving multiplying and adding, including using the distributive law to multiply two digit numbers by one digit, integer scaling problems and harder correspondence problems such as n objects are connected to m objects.**
- **Convert between different units of measure (e.g. km to m) -** use multiplication to convert from larger to smaller units
- *Understand the relation between non-unit fractions and multiplication/division of quantities. With particular emphasis on tenths and hundredths*
- *relate area to arrays and multiplication.*
- Problem solving work can involve finding all possibilities and combinations drawing on knowledge of multiplication tables facts
- *Pupils understand and use a greater range of scales in their representations* (Statistics)

Year 5 Multiplication

Informal methods to support mental Calculations

- multiply and divide numbers mentally drawing upon known facts
- multiply and divide whole numbers and those involving decimals by 10, 100 & 1000
- Recognise and use square & cube numbers (& notation)

Pupils should be taught throughout that percentages, decimals and fractions are different ways of expressing proportions.

24 x 15 = ?

I did: 24 × 5 = 120 (half of 24 × 10), then multiplied 120 by 3 to get 360

I did: (24 × 10) + (24 × 5).

Example of constructing equivalence statements:
4 x 35 = 2 x 2 x 35;
3 x 270 = 3 x 3 x 9 x 10
= 92 x 10

Spider diagrams

42 ÷ 6 =
0.7 × 6 = 4.2
0.07 × 6 =
4.2 ÷ 6 =
7 × 6 = 42
0.007 × 6 =

To be successful at multiplying decimal numbers using a written method, children need to be completely secure in using known multiplication facts to derive linked decimal facts. Spider diagrams provide a visual way of recording these facts.

Written Calculations

- multiply numbers up to 4 digits by a one- or two-digit number using a formal written method, including long multiplication for two-digit numbers

24 × 16 becomes

```
    2
    2 4
  × 1 6
  2 4 0
  1 4 4
  3 8 4
```

Ans

124 × 26 becomes

```
  1 2
  1 2 4
×   2 6
2 4 8 0
  7 4 4
3 2 2 4
1 1
```

Answer: 3224

124 × 26 becomes

```
  1 2
  1 2 4
×   2 6
  7 4 4
2 4 8 0
3 2 2 4
1 1
```

Answer: 3224

2741 × 6 becomes

```
  2 7 4 1
×       6
1 6 4 4 6
    4 2
```

Answer: 16 446

Compact methods for multiplication are efficient but often do not make the value of each digit explicit. When introducing multiplication of decimals, it is sensible to take children back to an expanded form such as the grid method where the value of each digit is clear, to ensure that children understand the process.

Does your answer seem reasonable?

Revert to expanded methods if children find formal calculation method difficult (see Y3/Y4)

Year 5 Multiplication

Representations to support mental and written calculations.

	3000	500	60	7		
20	60000	10000	1200	140		71340
4	12000	2000	240	28		14268
					Total	85608

What is the same and what is different about these two methods?

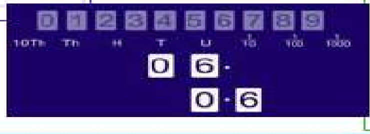
3567
x24
14268
71340
85608

To start multiplying using the **least significant digit** for the grid method will support children with implementation of the written procedure

Build on children's understanding: demonstrate multiplication of a decimal number alongside its whole number equivalent

326	3.26
× 8	× 8
2400	24.00
160	1.60
48	0.48
2608	26.08

0	6	12	18	24	30	36	42	48	54	60
0	0.6	1.2	1.8	2.4	3	3.6	4.2	4.8	5.4	6



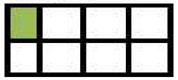
Fractions

- **multiply proper fractions and mixed numbers by whole numbers, supported by materials and diagrams**
- **identify, name and write equivalent fractions of a given fraction, represented visually, including tenths and hundredths**

Pupils connect multiplication by a fraction to using fractions as operators (fractions of), and to division, building on work from previous years. This relates to scaling by simple fractions, including fractions > 1.

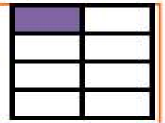
¼ x ½

Scaling by ½
"finding a half of a quarter"



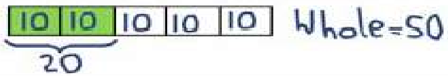
½ x ¼

"¼ of a ½": find a ½, then divide it by 4.



Encourage children to draw diagrams to represent situations or problems involving fractions
Model how to do this, for example:

$^2/_5$ of a number is 20. What is the number?



Links from other strands

- **identify multiples & factors, including finding all factor pairs of a number, & common factors of two numbers**
- **know and use the vocabulary of prime numbers, prime factors and composite (non-prime) numbers**
- **establish whether a number up to 100 is prime and recall prime numbers up to 19**
- **solve problems involving multiplication and division including using their knowledge of factors and multiples, squares and cubes, and including understanding the meaning of the equals sign**
- **solve problems involving multiplication and division, including scaling by simple fractions and problems involving simple rates**
- **use all four operations to solve problems involving measure [for example, length, mass, volume, money] using decimal notation, including scaling.**
- **convert between different units of metric measure; problems including money,.**

Other links: ratio,

Pupils use their knowledge of place value and multiplication and division to convert between standard units.
Pupils calculate the perimeter of rectangles and related composite shapes, including using the relations of perimeter or area to find unknown lengths. Missing measures questions such as these can be expressed algebraically, for example 4 + 2b = 20 for a rectangle of sides 2 cm and b cm and perimeter of 20cm.
Pupils calculate the area from scale drawings using given measurements.

Year 6 Multiplication

Informal methods to support mental Calculations	• **perform mental calculations, including with mixed operations and large numbers** *(increasingly large numbers & more complex calculations)* • *use all the multiplication tables to calculate mathematical statements in order to maintain fluency.* • **use estimation to check answers to calculations & determine, in the context of a problem, an appropriate degree of accuracy.** • **identify the value of each digit in numbers given to three decimal places and multiply and divide numbers by 10, 100 and 1000 giving answers up to three decimal places.** Use mental strategies to solve problems e.g. • x4 by doubling and doubling again • x5 by x10 and halving • x20 by x10 and doubling • x9 by multiplying by 10 and adjusting • x6 by multiplying by 3 and doubling Children should know the square numbers up to 12 × 12 & derive the corresponding squares of multiples of 10 e.g. 80 × 80 = 6400 How many different x/÷ facts can you make using 72? 7.2? 0.72? What is the best approximation for 4.4 x 18.6?
Written Calculations	• **multiply multi-digit numbers up to 4 digits by a two-digit whole number using the formal written method of long multiplication** *(short & long multiplication)* • **multiply one-digit numbers with up to two decimal places by whole numbers** $$\begin{array}{lr} \pounds & 6.23 \\ \times & 27 \\ \hline & 43.61 \\ & 1^{1}2^{2}4.60 \\ \hline \pounds & 168.21 \\ \hline & {}_{1}\end{array}$$ **Revert to expanded methods if children find formal calculation method difficult (see Y4/Y5)**

Year 6 Multiplication

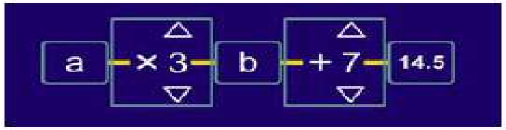
Representations to support mental and written calculations.

Look at long-multiplication calculations containing errors, identify the errors and determine how they should be corrected

x	8	0.4	0.06	
11	88	4.4	0.66	= 93.06

8.46
X 11
93.06

What's the same?
What's different?

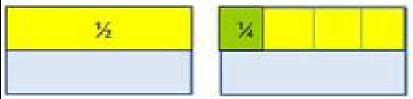


Fractions

- multiply simple pairs of proper fractions, writing the answer in its simplest form e.g. ¼ x ½ = 1/8

Three key applications of understanding:

- Recognise that ¼ of 12, ¼ x 12 and 12 divided by 4 are equivalent
- Use cancellation to simplify the product of a fraction and an integer e.g. ⅕ x 15 = 3, ⅖ x 15 = 2 x ⅕ x 15 = 2x3 = 6
- Work out how many ⅕s in 15, how many ⅖s in 15, how many 2/5s in 1 etc.



To calculate ¼ x ½ , find ½ of a rectangle/array, then divide that ½ into ¼s. So ¼ of ½ is 1/8

Pupils should use a variety of images to support their under-standing of multiplication with fractions. This follows earlier work about fractions as operators (fractions of), as numbers, and as equal parts of objects, e.g. as parts of a rectangle.

Links from other strands

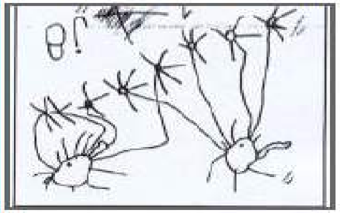
- **identify common factors, common multiples and prime numbers**
- **use their knowledge of the order of operations to carry out calculations involving the four operations**
- **solve problems involving addition, subtraction, multiplication and division**
- *explore the order of operations using brackets; for example, 2 + 1 x 3 = 5 and (2 + 1) x 3 = 9.*
- **Fractions, decimals and percentages** including equivalences in different contexts.
- **solve problems involving the relative sizes of two quantities where missing values can be found by using integer multiplication and division facts**
- **solve problems involving the calculation of percentages [for example, of measures, and such as 15% of 360] and the use of percentages for comparison**
- **solve problems involving similar shapes where the scale factor is known or can be found**
- **solve problems involving unequal sharing and grouping using knowledge of fractions and multiples.**
- **Algebra** including formulae, linear number sequences, combinations of variables
- **Measurement** including solving problems with conversion of units, decimal notation, area & volume
- **Statistics** including pie charts, line charts and calculating the mean

Year 1 Division

Mental Calculations

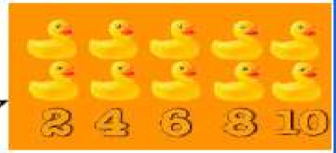
Solve one-step problems involving multiplication and division, by calculating the answer using concrete objects, pictorial representations and arrays with the support of the teacher.

(Pupils) make connections between arrays, number patterns, and counting in twos, fives and tens.



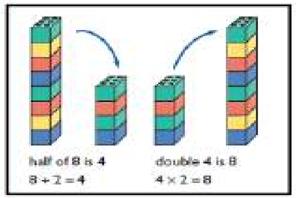
Count on or back in 2s, 5s and 10s and look for patterns.

Songs are useful for counting in steps.

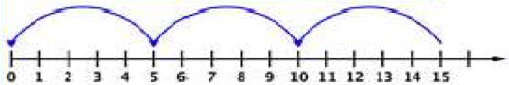


Written Calculations

Pictorial jottings to support the calculation of 8 ÷ 4



Children should experiment with the concepts of sharing and grouping in a number of contexts. Initially they use their own recording—moving towards fluent, symbolic notation in Year 2. Conceptual understanding and recording should be continuously supported by the use of **arrays** as a default model, as well as other representations, (see below.)



The relationship between multiplication and division must be continually considered.

Fractions

Recognise, find and name a half as one of two equal parts of an object, shape or quantity
Recognise, find and name a quarter as one of four equal parts of an object, shape or quantity. (*See* Representations *above.)*

Links from other strands

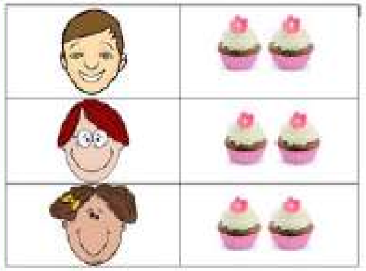
They practise counting as reciting numbers and counting as enumerating objects, and counting in twos, fives and tens from different multiples to develop their recognition of patterns in the number system (for example, odd and even numbers). (PLACE VALUE).
Pupils are taught half and quarter as 'fractions of' by solving problems using shapes, objects and quantities. (FRACTIONS)

Year 1 Division

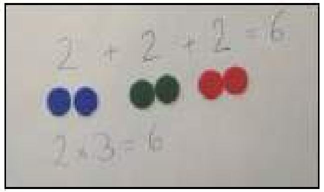
Representations to support mental and written calculations.

Use a range of concrete and pictorial representations, including:

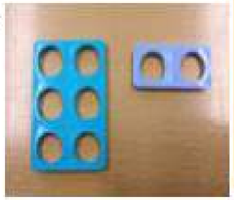
- Manipulatives to support children's own recording; and understanding of *sharing* and the link with multiplication.
  "How can we share 6 cakes between 2 people?"



Here, the cakes are placed in an array formation.



How many 2 tiles can we fit on the 6 tile?

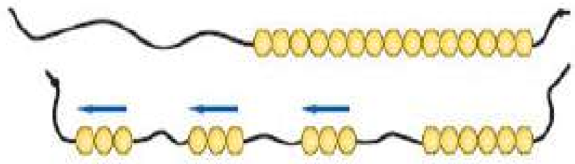


Moving from concrete to pictorial, counters represent the cakes to reinforce the relationship between multiplication and division.

- Manipulatives, and real-life objects to support children's own recording; and understanding of *grouping* and the link with multiplication.

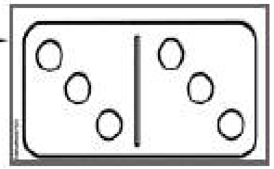
Coat hangers and socks support calculation of 8÷2

Bead strings



15 ÷ 2 using grouping model

"Double 3 is 6. Half of 6 is 3."



- Dominoes and dice to reinforce concepts of doubling and halving.

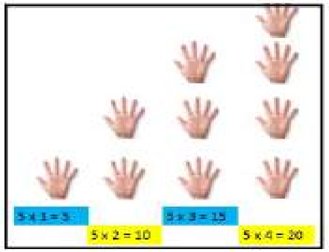
Year 2 Division

Mental Calculations

The relationship between multiplication and division must be continually considered.

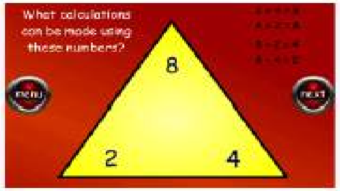
- **Recall and use multiplication and division facts for the 2, 5 and 10 multiplication tables, including recognising odd and even numbers .**
- **Calculate mathematical statements for multiplication and division within**
- **the multiplication tables and write them using the multiplication (×), division (÷) and equals (=) signs .**

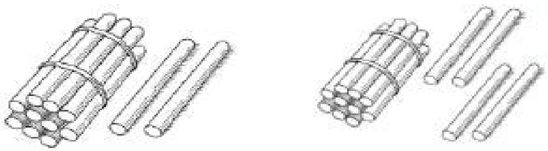
"5, one time", "5, two times" and so on.



Written Calculations

- **Show that multiplication of two numbers can be done in any order (commutative) and division of one number by another cannot**
- **Solve problems involving multiplication and division, using materials, arrays, repeated addition, mental methods, and multiplication and division facts, including problems in contexts. (*See below.)***





½ of 26 = 13
26 ÷ 2 = 13

Pupils decode a problem first, represent it using manipulatives and jottings; and finally record it symbolically.

Fractions

Recognise, find, name and write fractions ⅓, ¼, ¾, 2/4 of a length, shape, set of objects or quantity
Write simple fractions for example, ½ of 6 = 3 and recognise the equivalence of ½ and 2/4.

Year 2 Division

Representations to support mental and written calculations.

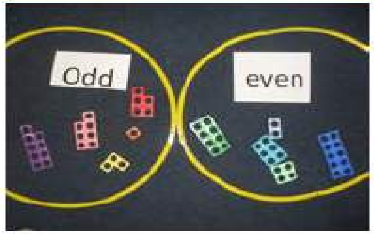
Use a range of concrete and pictorial representations, including:

- Arrays

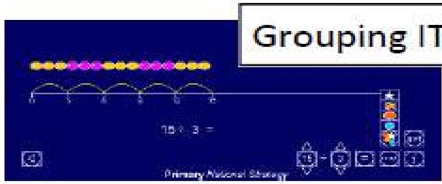
$7 \times 2 = 14$
$14 \div 2 = 7$

$2 \times 7 = 14$
$14 \div 7 = 2$

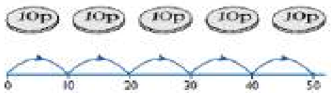
Is 14 an odd number? How do you know?



Grouping ITP



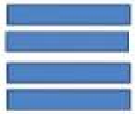
- Number lines to support grouping



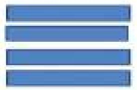
10p + 10p + 10p + 10p + 10p = 50p
10p × 5 = 50p
5 hops of 10

"How many groups of 5 minutes have passed when the minute hand reaches twenty past?"

- Representations to support multiplicative reasoning:



Using Dienes: *"If 40 ÷ 10 = 4 and 30 ÷ 10 = 3, what do you think 70 ÷ 10 would be? Why?"*



Links from other strands

- **Count in steps of 2, 3, and 5 from 0, and in tens from any number, forward and backward.**
- **Recognise the place value of each digit in a two-digit number (tens, ones) (PLACE VALUE).**
- **Tell and write the time to five minutes, including quarter past/to the hour and draw the hands on a clock face to show these times, (MEASURES).**

Year 3 Division

Mental Calculations

Pupils should be taught to recall and use multiplication and division facts for the 3, 4 and 8 multiplication tables.

Pupils continue to practise their mental recall of multiplication tables… in order to improve fluency. Pupils develop efficient mental methods, for example, using commutativity and associativity (e.g., 4 × 12 × 5 = 4 × 5 × 12 = 20 × 12 = 240) and multiplication and division facts to derive related facts.

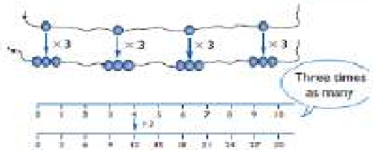
36 ÷3 = 12

30 6

30 ÷3=10 6 ÷3=2

\+

"4 x 3 is 12, so 12 ÷ 3 = 4."



Written Calculations

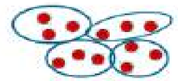
Pupils should be taught to:

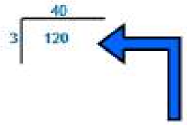
•write and calculate mathematical statements for multiplication and division using the multiplication tables that they know, including for two-digit numbers times one-digit numbers, using mental and progressing to formal written methods.

•solve problems, including missing number problems, involving multiplication and division, including positive integer scaling problems and correspondence problems in which n objects are connected to m objects, (see <u>Links from other strands</u>, below.)

"I know 6÷3=2,
so 60÷3=20."
"I know 12÷3=4,
so 120÷3=40."

120 ÷ 3

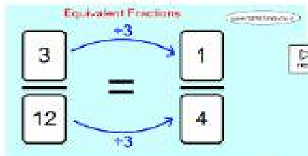


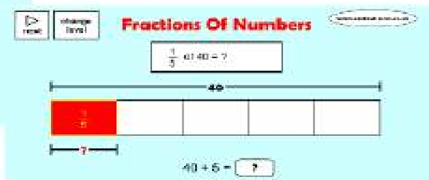


New written methods can be modelled alongside mental or informal methods to ensure understanding.

Fractions

- **Recognise that tenths arise from dividing an object into 10 equal parts and in dividing one-digit numbers or quantities by 10.**
- **Recognise and show, using diagrams, equivalent fractions with small denominators.**
- **Recognise, find and write fractions of a discrete set of objects: unit fractions and non-unit fractions with small denominators.**

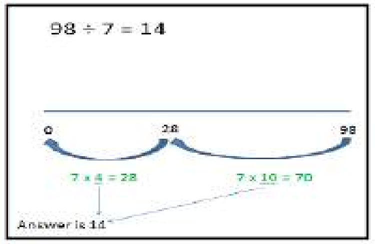




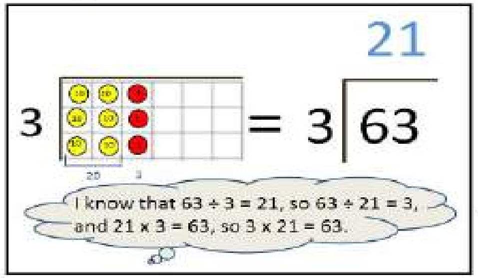
Year 3 Division

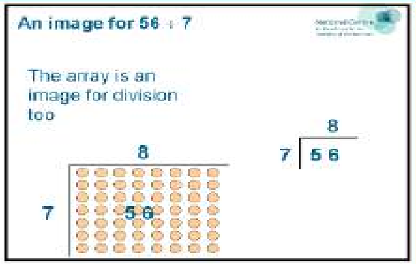
Representations to support mental and written calculations.

Use a range of concrete and pictorial resources, including:



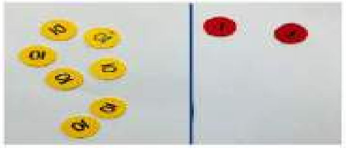
63 ÷ 3 equals three groups of 2 tens and a one.

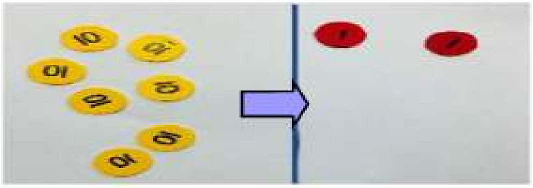


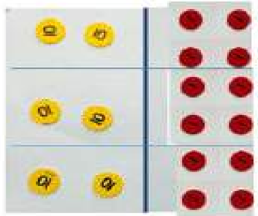


How could I calculate 72÷3 ?

Informal exploration with manipulatives supports the progression to formal written methods—which is continued in Year 4.

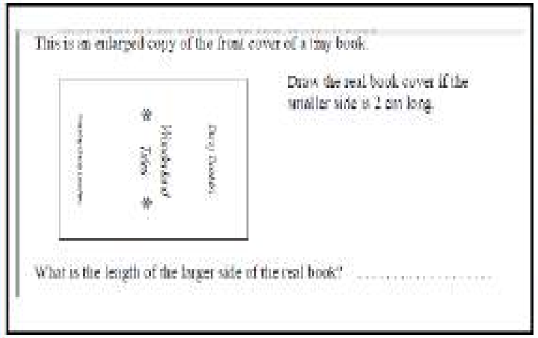






$$3\overline{)72}\quad\text{answer } 24$$

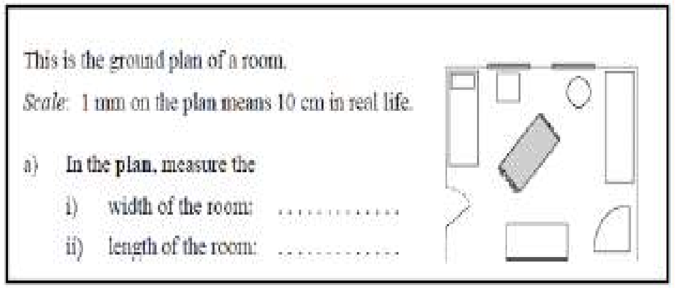
Links from other strands


This is an enlarged copy of the front cover of a tiny book.

Draw the real book cover if the smaller side is 2 cm long.

What is the length of the larger side of the real book? ....

Pupils solve simple problems in contexts, including measuring and scaling contexts, (e.g., four times as high etc.) and correspondence problems.


This is the ground plan of a room.

Scale: 1 mm on the plan means 10 cm in real life.

a) In the plan, measure the

i) width of the room: ....

ii) length of the room: ....

Year 4 Division

Informal methods to support mental Calculations

Pupils should be taught to:

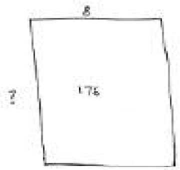
- **recall multiplication and division facts for multiplication tables up to 12 × 12**
- **use place value, known and derived facts to multiply and divide mentally,**

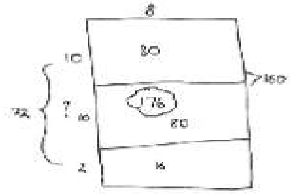
including: multiplying by 0 and 1; dividing by 1; multiplying together three numbers

- **recognise and use factor pairs and commutativity in mental calculations**

I know that 6÷3=2, so 600÷3=2.

Using known facts and blank arrays to calculate 176÷8.





176÷8 = 22

Pupils practise mental methods and extend this to three-digit numbers to derive facts.

Written Calculations

Pupils should be taught to:

- **multiply two-digit and three-digit numbers by a one-digit number using formal written layout**
- **solve problems involving multiplying and adding, including using the distributive law to multiply two digit numbers by one digit, integer scaling problems and harder correspondence problems such as n objects are connected to m objects.**

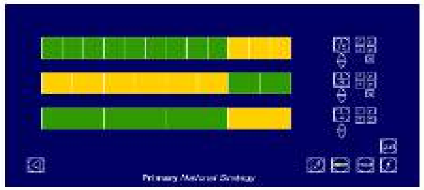
Pupils practise to become fluent in the formal written method of short multiplication and short division with exact answers .

Revert to expanded methods if children find formal calculation method difficult

Fractions

Pupils should be taught to:

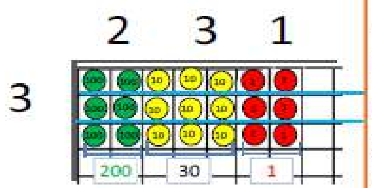
- ***recognise and show, using diagrams, families of common equivalent fractions***
- ***recognise that hundredths arise when dividing an object by one hundred and dividing tenths by ten.***
- ***solve problems involving increasingly harder fractions to calculate quantities, and fractions to divide quantities, including non-unit fractions where the answer is a whole number***
- ***find the effect of dividing a one- or two-digit number by 10 and 100, identifying the value of the digits in the answer as ones, tenths and hundredths***



Year 4 Division

Representations to support mental and written calculations.

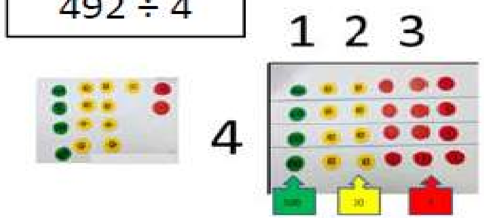
693 ÷ 3



Children can work in pairs: child A constructs the array (dividing manipulatives into 3 rows), child B checks it and records this in a formal, short division format.

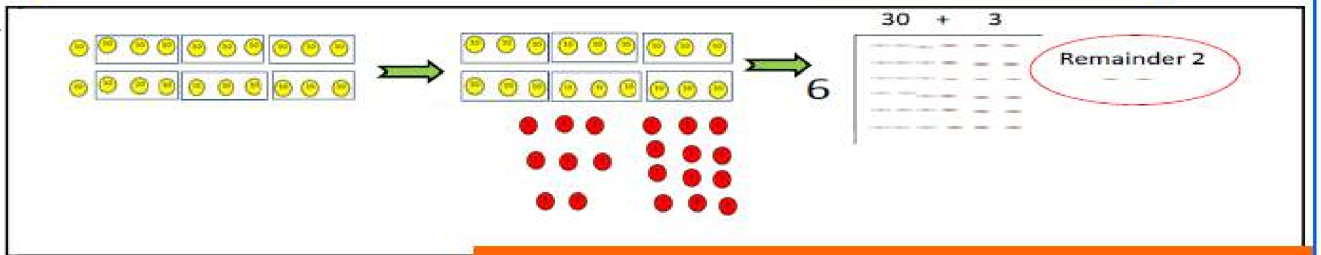
By working through larger number calculations with manipulatives, children gain experience of ex-change (re-partitioning) within division algorithms.

492 ÷ 4



By the end of Year 4, children need to have encountered remainders in a number of contexts. Pupils can be introduced to remainders using known facts: e.g. 13÷4; and then progress to larger numbers. (See below).

200 ÷ 6 = 33 r.2



Money can be used instead of place value counters.

Links from other strands

- Convert between different units of measure [for example, kilometre to metre; hour to minute]
- Estimate, compare and calculate different measures, including money in pounds and pence (MEASURES)
- Recognise that hundreths arise when dividing an object by one hundred and dividing tenths by ten. (FRACTIONS)

Year 5 Division

Informal methods to support mental Calculations

. Pupils should be taught to:

- multiply and divide whole numbers and those involving decimals by 10, 100 and 1000
- multiply and divide numbers mentally drawing upon known facts

identify multiples and factors, including finding all factor pairs of a number, and common factors of two numbers .

If $42 \div 6 = 7$

$\div 10$ $\div 10$

Then $4.2 \div 6 = 0.7$

Number lines

$400 \div 17 = 23$ r 9

Factorising

$480 \div 15$

$= 480 \div 5 \div 3$

"I know that the answer to $138 \div 6$ will be close to 20, because $2 \times 6 = 12$, so $20 \times 6 = 120$."

Pupils apply all the multiplication tables and related division facts frequently and use them confidently .

Written Calculations

Pupils practise and extend their use of the formal written methods of short multiplication and short division.

- **Divide numbers up to 4 digits by a one-digit number using the formal written method of short division and interpret remainders appropriately for the context.**

98 ÷ 7 becomes	432 ÷ 5 becomes	496 ÷ 11 becomes
1 4	8 6 r 2	4 5 r 1
7 | 9 ^{2}8	5 | 4 3 ^{3}2	1 1 | 4 9 ^{5}6
Answer: 14	Answer: 86 remainder 2	Answer: $45 \frac{1}{11}$

- **Pupils interpret non-integer answers to division by expressing results in different ways according to the context, including with remainders, as fractions, as decimals or by rounding. (See Representations below.)**

Revert to expanded methods if children find formal calculation method difficult

Fractions

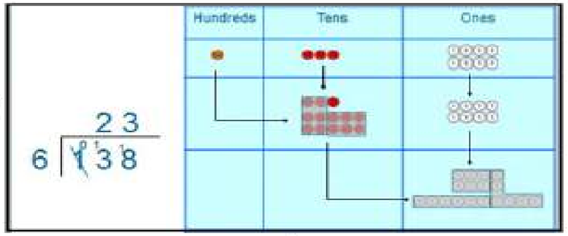
- **Recognise mixed numbers and improper fractions and convert from one form to the other and write mathematical statements > 1 as a mixed number .**
- ***Pupils connect equivalent fractions > 1 that simplify to integers with division and other fractions > 1 to division with remainders.***
- ***Pupils connect multiplication by a fraction to using fractions as operators (fractions of), and to division.***
- ***Pupils should make connections between percentages, fractions and decimals***

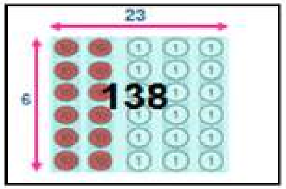
Year 5 Division

Representations to support mental and written calculations.

Can we divide this 100 token into 6 equal groups?, then we must exchange it for ten 10 tokens. Can we divide into 6 groups now?

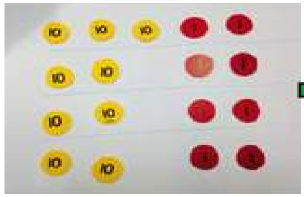
Short division with exchange.

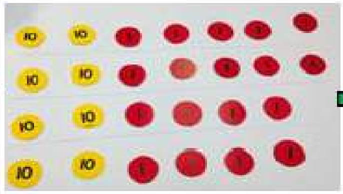


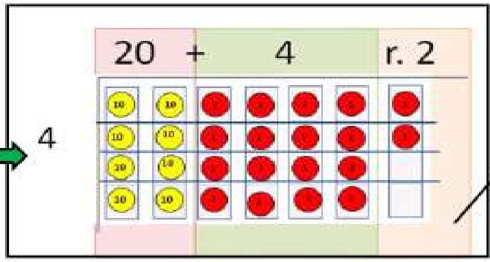


Practical experience with manipulatives is vital for children to talk through the language of division e.g. *exchange*, *remainder*; and to embed conceptual understanding.

Understanding remainders.







2 out of a whole group of 4 = $\frac{2}{4} = \frac{1}{2} = 0.5$

$98 \div 4 = \frac{98}{4} = 24 \text{ r } 2 = 24\frac{1}{2} = 24.5$

What is the same? What's different about the ways that these remainders are expressed?

Links from other strands

- ***Pupils use all four operations in problems involving time and money, including conversions. …..using decimal notation, including scaling.***
- ***calculate and compare the area of rectangles (including squares). (MEASURES)***

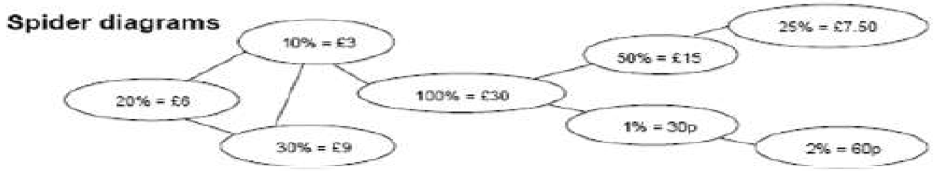
- establish whether a number up to 100 is prime and recall prime numbers up to 19
- recognise and use square numbers and cube numbers, and the notation for squared (2) and cubed (3)
- solve problems involving multiplication and division including using their knowledge of factors and multiples, squares and cubes and including scaling by simple fractions and problems involving simple rates.
- solve problems involving addition, subtraction, multiplication and division and a combination of these, including understanding the meaning of the equals sign. (NUMBER—MULTIPLICATION AND DIVISION)

Year 6 Division

Informal methods to support mental Calculations

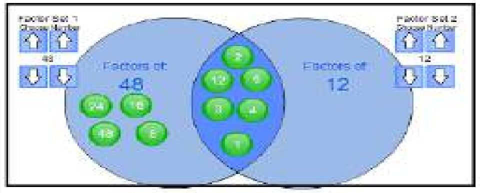
. Pupils should be taught to:

- perform mental calculations, including with mixed operations and large numbers.
- use their knowledge of the order of operations to carry out calculations involving the four operations.
- identify common factors, common multiples and prime numbers.



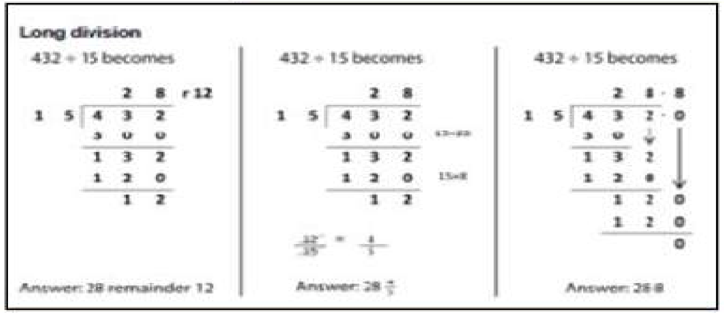
I know that 366 will divide by 6 because it has 2 and 3 as factors

- Solve problems involving addition, subtraction, multiplication and division
- use estimation to check answers to calculations and determine, in the context of a problem, an appropriate degree of accuracy.



Written Calculations

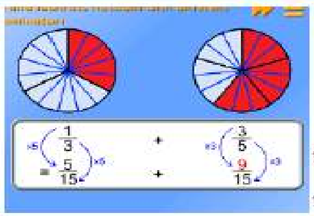
- divide numbers up to 4 digits by a two-digit whole number using the formal written method of long division, and interpret remainders as whole number remainders, fractions, or by rounding, as appropriate for the context
- divide numbers up to 4 digits by a two-digit number using the formal written method of short division where appropriate, interpreting remainders according to the context.
- *Pupils practise division for larger numbers, using the formal written methods of short and long division.*

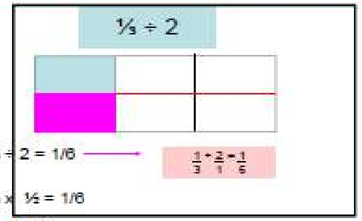


Revert to expanded methods if children find formal calculation method difficult

Fractions

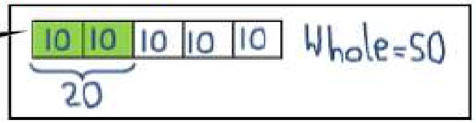
- use common factors to simplify fractions,
- compare and order fractions, including fractions > 1
- add and subtract fractions with different denominators and mixed numbers, using the concept of equivalent fractions
- divide proper fractions by whole numbers [for example, 1/3 ÷2 = 1/6 .]
- associate a fraction with division and calculate decimal fraction equivalents [for example, 0.375.]





- *Pupils use their understanding of the relationship between unit fractions and division to work backwards.* use written division methods in cases where the answer has up to 2 dp.

2/5 of a number is 20.
What is the number?



Videos to Support Mathematical Teaching and Learning

Multiplication https://www.ncetm.org.uk/resources/40530 KS1 - Multiple Representations of Multiplication KS1- The commutative law for multiplication Lower KS2 - Grid multiplication as an interim step Upper KS2 - Moving from grid to a column	**Algebra** https://www.ncetm.org.uk/resources/43649 KS1 - Look at 'missing numbers' KS2 - Equations and substitution KS3 - Factorising*	**Number facts** https://www.ncetm.org.uk/resources/40533 KS1 - Number bonds to ten KS1 - Consolidation and practice (Addition and Subtraction) KS1 - Reinforcing Table Facts KS1 - Rapid recall of multiplication facts	**Division** https://www.ncetm.org.uk/resources/43589 KS1- Sharing and grouping KS 2 - Place value counters for division KS 3 - Group working on problems*
Number and Place value https://www.ncetm.org.uk/resources/40534 KS1 - Counting in steps of one and ten KS1 - Partitioning in different ways KS1 - Addition and Subtraction KS1 - Using resources to develop fluency and understanding KS2 - Partitioning (subtraction)	**Fractions** https://www.ncetm.org.uk/resources/43609 KS1 - Adding fractions and mixed numbers KS2 - Using an array to add fractions KS2 - Bar model dividing by fractions KS3 - Fraction wall to add fractions*	**Subtraction** https://www.ncetm.org.uk/resources/40532 Lower KS2 – Partitioning Lower KS2 - Discussing Subtraction Strategies Lower KS2 - Developing Column Subtraction Upper KS2- Column Subtraction	**Multiplicative reasoning** https://www.ncetm.org.uk/resources/43669 KS2 - Bar model for multiplication KS3 - Ratio and proportion*

Glossary

Add +	Add, total, sum, more, plus, increase, altogether
Algebra	a symbol representing a number
Arrays	A rectangular representation where each row and column must have the same number of objects or pictures.
Commutative	In addition, numbers can be added up in any order and the total remains the same e.g. a + b = b + a.
Divide ÷	Dividing is a quick way of subtracting several lots of the same number of quantity, or splitting it up into equal groups.
Divisor	The divisor is the number you divide by e.g. in 6÷3 the divisor is 3.
Equals	the same value as, equivalent, balance
Equation	A number sentence that uses letters or symbols to replace digits, a statement where two mathematical expressions have the same value.
Generalise	Look for a general pattern that will help to solve a related problem.
Grid	A way to organise a multiplication or division calculation where the number being divided is partitioned. The partial products are shown in the grid. See diagram in multiplication and division sections.

Glossary

Grouping	Where a set of objects or a number is grouped into an already established number until no more groups can be made. E.g. 35 sweets shared between 7 friends would make 7 groups of 5.
Inverse	Opposite or reverse operations, e.g. 16 – 7 = 9 so 9 + 7 = 16
Multiply x	So many groups of, lots of, and sets of. Times, find the product of.
Pedagogy	Teaching method used e.g. asking questions, encouraging to look for patterns etc.
Quotient	A quotient is the whole number of times you can divide one number by a number.
Remainder	If you can't divide a number exactly you have an amount left over – this is called the remainde--r.
Repeated addition	Repeated addition is the process of grouping. Where a number is repeatedly added from 0 to the target number e.g. repeatedly adding 5. For larger numbers, multiples of e.g. 5 can be repeatedly added.
Repeated subtraction	Repeated subtraction is the process of grouping. Where a number is repeatedly subtracted from the total e.g. repeatedly subtracting 5 from 35 (7 times). For larger numbers, multiples of e.g. 5 can be repeatedly subtracted.
Sharing	Where a set of objects or a number is shared equally into a given number of sets. E.g. 28 sweets are shared into 7 equal piles.
Subtract –	Minus, take away, find the difference, count how many left, find that many fewer than before.